Path delay model based on alpha-stable distribution for the 60 GHz indoor channel

Nourddine Azzaoui, Laurent Clavier, Rachid Sabre

Path delay model based on α-stable distribution for the 60GHz indoor channel.

Nourddine Azzaoui
ENESAD, Université de Bourgogne
26,BD docteur Petitjean
21000, Dijon
Email: n.azzaoui@enesad.fr

Laurent Clavier
ENIC, I.E.M.N - UMR.CNRS 8520
Cité scientifique, rue Guglielmo Marconi
59658 Villeneuve d'Ascq Cedex
Email: clavier@enic.fr

Rachid Sabre
ENESAD, Université de Bourgogne
26,BD docteur Petitjean
21000, Dijon
Email: r.sabre@enesad.fr

***Abstract*— In this work we give a statistical model of the paths' arrival time in the 60GHz wide band channel. This model is based on the class of stable distributions, we show its accurateness on modeling real data. We also give a theoretical justification of its compatibility.**

I. Introduction

High data rate transmission in an indoor environment is an important issue for the next generation of wireless communication systems. For high data rate links (155 Mbits/s), the spectrum around 60 GHz is an attractive solution [1]. A massive amount of spectral space (2 to 5 GHz) has been allocated world-wide for unlicensed, dense wireless local communications. At this frequency, the signal is strongly attenuated and the propagation characteristics are different from microwaves since the molecules of oxygen in the atmosphere interact with the radio wave. The peak value of the absorption attenuation coefficient reaches about 15 dB/km at 60 GHz. This property, which appears to be a disadvantage for wide range radio transmission, is an advantage for indoor cellular systems. It provides an important interference reduction, simplifying the frequency planning and allowing an efficient use of the spectrum. Furthermore, the millimeter wave band facilitates miniaturization of components and antennas. The use of full monolithic microwave integrated circuit (MMIC) transmitters and receivers will make possible the use of hand held multimedia terminals.

However, in the case of mobile indoor radio communications, multipath propagation causes severe degradations of the transmission quality. To solve this problem and hence prepare the future generation of wide band indoor networks, an accurate and flexible modelling of the channel is necessary. This will allow realistic simulations and the optimization of the communication chain.

The channel can be represented by a linear filter characterized by its impulse response. Extensive works about modelling the impulse response was made in the literature [2], [3], [4]. The path arrival times is one difficult aspect to model. If it is tempting to describe them in terms of a Poisson distribution, it was shown in [5] that, in the case of 60GHz channel in small rooms, this modelling is not accurate. Turin [1], also observing that the Poisson hypothesis was not confirmed by experience, showed in the case of 488, 1280 and 2980MHz that times of arrival could be modelled by a modified Poisson process, where the Poisson parameter changes when a path arrives. This model was further developed by Suzuki [3] and Hashemi [6]. It is termed the Δ k model and based on the fact that echoes arrive in clusters, corresponding to reflections on closely spaced buildings. Saleh and Valenzuela [4] developed a different model in the case of the 7.5GHz indoor channel. They also considered the clusters and introduced two arrival time processes : one for the clusters and one for the echoes in each cluster.

In this paper we investigate the observed data of impulse responses collected in IEMN[1] [5]. We introduce a new distribution, which can be seen as an association of several Poisson processes, to give a new statistical model of the impulse response, especially the arrival times : the α-stable distributions.

In the first part we describe briefly some properties of the stable distribution and an historical review about estimating their parameters. In the second part we present the statistical results of fitting the delays to α-stable distributions. In the third section the proposition (V-A) gives a theoretical justification of modelling the delays by stable distributions and confirms the empirical results.

II. Description of Measurement System.

A wide band channel sounder based on the measurement of the channel Transfer Function H(f) in the frequency domain [7] has been developed in IEMN. The measured sampled frequency response of the multipath radio channel is converted into the impulse response in the time domain by taking the inverse Fourier transform. The frequency step of 1.25 MHz yields a maximum measurable delay of 800 ns. The frequency span of 2 GHz yields a delay resolution of 0.5 ns. The measurement system has been dimensioned in such a way that the signal-to-noise ratio (SNR) at the network analyser was not less than 10 dB. The apparatus dynamic was of 124 dB. Furthermore, a processing dynamic of 30 dB from the strongest detected path was chosen to obtain reliable results. A transmitted power of 10 dBm was sufficient to guarantee this measurement quality on a TR distance that can reach 45

[1]Institut d'Electronique, de Microélectronique et de Nanotechnologies

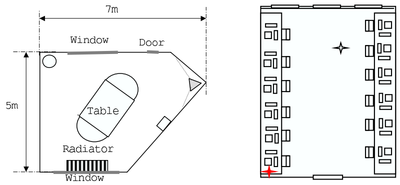

Fig. 1. measurements rooms

m. In our measurements, the environment was kept static. Only the receiver was moved between two measurements by using an automated positioning system that consists in a linear table of 50 cm with a step-motor driven millimeter screw along it. Measurement campaigns were made in rooms presented on [fig 1] using a 60 GHz channel sounder based on a network analyser [5, 7]. Respectively 1140 and 6500 impulse responses were measured in the rooms.

III. ALPHA STABLE DISTRIBUTIONS

The α-stable distribution is a direct generalization of the gaussian distribution and shares many of it's familiar properties :

- The convolution stability property, which means that the convolution of two stable distributions is also stable, or in term of random variables the sum of two stable random variables is also a stable one.
- The central limit theorem, which means that every stable random variable may be expressed as a limit, in distribution, of a normalized sum of independents and identically distributed random variables.

Besides, they are parametric distributions, because they are fully described by four parameters.

Since their discovery by Paul Levy in 1925, a vast amount of knowledge has been accumulated about the properties of these probability distributions. On the other hand they have been found to provide useful models for various application fields, especially phenomena with large fluctuations and high variability that are not compatible with the Gaussian models.

Except the Gaussian , the cauchy and the Levy distributions which are special cases of the stable class, there is no exact expression of the probability density function of an $\alpha-$stable distribution. However we can approximate it through its characteristic function which is given by :

$$\phi(\theta) = \left\{ \begin{array}{c} \exp\{-\sigma^{\alpha}|\theta|^{\alpha}(1 - i\beta \mathrm{sign}(\theta)\tan\frac{\pi\alpha}{2}) + i\mu\theta\} \\ if \alpha \neq 1 \\ \exp\{-\sigma|\theta|(1 + i\beta\frac{2}{\pi}\mathrm{sign}(\theta)\ln|\theta|) + i\mu\theta\} \\ if \alpha = 1 \end{array} \right. \tag{1}$$

where

$$\mathrm{sign}(\theta) = \left\{ \begin{array}{ccc} 1 & if & \theta > 0 \\ 0 & if & \theta = 0 \\ -1 & if & \theta < 0 \end{array} \right.$$

and α, σ, β and μ are the four parameters characterizing the stable distribution.

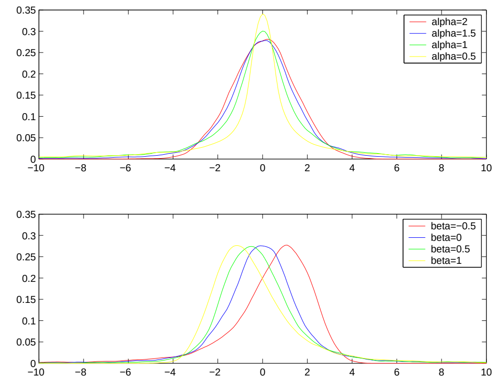

Fig. 2. the probability density function of a stable distribution with $\mu = 0$, $\sigma = 1$ and different values of α and β

- α is called the characteristic exponent $(0 < \alpha \leq 2)$: it measures the thickness of the tail of the distribution. Thus the larger the value of α the less likely it is to observe values which are far from the central location.
- μ is the location parameter $(-\infty < \mu < \infty)$: for instance, in an observed sample the most observations are concentrated about its value. It corresponds to the mean for $1 < \alpha \leq 2$ and to the median for $0 < \alpha \leq 1$
- σ is the dispersion parameter $(\sigma > 0)$: it is like the standard error in the case of a gaussian distribution.
- β is the index of symmetry $(-1 \leq \beta \leq 1)$ which characterizes the dissymmetry of the density function about its central location. When $\beta = 1$ we say that the distribution is totally skewed to the right; it is symmetric if $\beta = 0$.

[fig 2] presents the density function obtained for different values of the parameters α and β and illustrates their effect on the behavior and the form of an α-stable distribution.

In practice it is very important to estimate the parameters of a stable distribution from an observed sample especially the index α and the scale parameter σ. Several methods have been proposed in the literature. Among them, maximum likelihood method developed by DuMouchel [8] is asymptotically efficient but difficult to compute. Zolotarev [9] estimates the parameters by the method of moments but requires that the location parameter is known. McCulloch [10] generalized the Famma and Roll [11] method to provide a simple consistent estimator of the parameters. A characteristic function based method was introduced by Koutrevelis [12] by using a regression type estimation. This last method has the importance to be easy to compute and appears to be more accurate if no parameter is a priori known.

IV. Statistical Results

It was shown in [5] that, in the case of the 60GHz channel in small rooms, modelling the delays with a Poisson distribution is not accurate. From the different observed delays, we have noticed an important variability in the realization of arrival delays. We, also, observed the existence of many realization that are considered as outliers for the usual statistical models

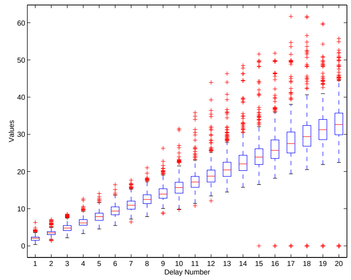

Fig. 3. Box-plot of the observed delays.

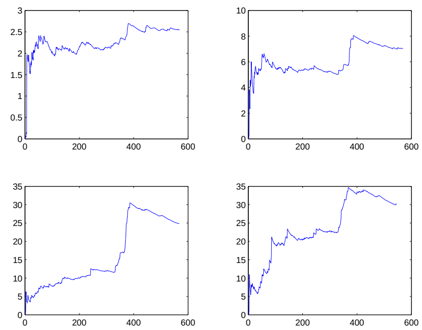

Fig. 4. Test for the infinite variance we present the partial variances of the delays samples.

: for example the box plot in [fig 3] shows clearly this fact, the (+) are outliers.

To confirm the high variability, we have performed the test for infinite variance, by the method presented in [13]. As it is shown by [fig 4], the variance of the delays may be considered as tending towards infinity. We have also made a kernel non parametric estimation of the densities of the delays, we obtained a bell shape curves skewed to the right see [fig 5] which confirms the dissymmetry remarked in the box plots.

This preliminary results show that the delays may be approximated by $\alpha-$stable random variables. To confirm the stable approach, we have estimated the four parameters characterizing the stable distribution from the delays samples using the Koutrouvelis [12] regression type method for its simplicity and its accurateness [14]. From this observed values we have

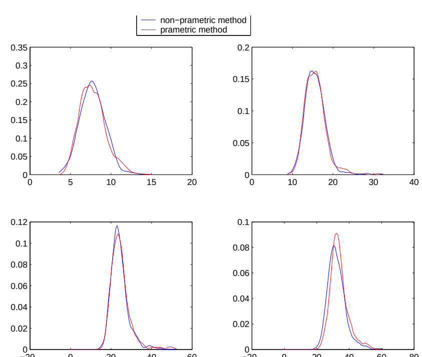

Fig. 5. Examples of estimated densities of the delays by the parametric and the non-parametric method.

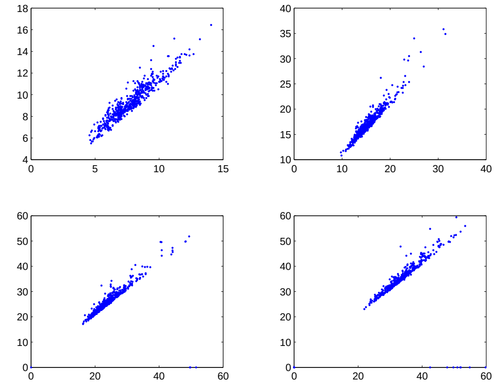

Fig. 6. clouds representing the dependence between consecutive delays

order of arrival	mean delay	order of arrival	mean delay
1	0	22	35.5060
2	1.9774	23	37.4239
3	3.4366	24	39.2968
4	4.9040	25	41.2802
5	6.3774	26	42.7980
6	7.9775	27	44.3326
7	9.5245	28	46.1994
8	11.0601	29	47.5919
9	12.6370	30	49.2257
10	14.2232	31	50.7094
11	15.9078	32	52.3222
12	17.5303	33	53.2318
13	19.2500	34	54.8898
14	21.0584	35	55.9073
15	22.8572	36	56.6969
16	24.7044	37	58.0895
17	26.5525	38	59.0987
18	28.4076	39	58.7361
19	30.3190	40	62.7386
20	32.0349	41	63.6094
21	33.6977	42	67.0000

Fig. 7. example of mean delays between arrivals observed in one room.

approximated the theoretical densities and plotted them in the same graphic with the densities obtained by the non parametric method. Thus from the [fig 5] it is clear that the $\alpha-$stable fits well the probability distribution of the arrival times in the 60GHz channel. This fact was confirmed by a Kolmogorov-Smirnov test performed on the observed and simulated data.

As it was first remarked, in the box plot, the means of the observed delays are increasing see also [fig 7]. A test of spearman shows that, in the case of indoor 60 GHz channel, in 72.5 percent of the observed impulse responses the delays are with independent increments. The [fig 6] shows the existence of a linear dependence between two consecutive delays. Assuming that the delays between paths are constant in a given room, and motivated by its simplicity and its low cost of computation we have adopted a linear model as follows :

$$Y = AX + \epsilon \quad (2)$$

where $Y = (\tau_2, \tau_3, ..., \tau_N)$, $X = (\tau_1, \tau_2, ..., \tau_{N-1})$, N is the number of arrivals, and ϵ is an α-stable white noise see [fig 8] for the estimation of its parameters. A is the matrix of the linear model parameters. Dependence between increment could be modelled by a diagonal matrix since we supposed that the delays are with independent increment. The estimation of A was made by the least absolute deviation method (LAD) (see [13] for its compatibility with α-stable models and its consistence). The [fig 9] represent on the same graphic the observed delays and those predicted using the linear model

Order of arrivals	Alpha	Beta	Sigma	mu
1	1.9288	1.0000	0.3558	0.0000
2	1.7530	1.0000	0.3536	-0.0000
3	1.8738	1.0000	0.3724	0.0000
4	1.8941	1.0000	0.4153	0.0000
5	1.8632	1.0000	0.3734	-0.0000
6	1.7502	1.0000	0.3493	-0.0000
7	1.7382	1.0000	0.3587	0.0000
8	1.7216	1.0000	0.3738	-0.0000
9	1.6371	1.0000	0.4081	-0.0000
10	1.6541	1.0000	0.3776	-0.0000
11	1.5710	1.0000	0.4183	0.0100
12	1.5488	0.9960	0.4392	-0.0000
13	1.5615	0.7069	0.4433	0.0137
14	1.4967	0.9095	0.4631	0.0051
15	1.2874	1.0000	0.4465	0.0041
16	1.6501	0.9214	0.5007	0.0042
17	1.6062	1.0000	0.5437	-0.0000
18	1.6090	1.0000	0.5416	0.0073
19	1.3609	1.0000	0.4376	0.0074
20	1.5749	1.0000	0.5343	0.0114
21	1.6067	1.0000	0.5817	0.0043
22	1.5774	1.0000	0.5725	0.0085
23	1.4423	1.0000	0.6110	0.0328
24	1.3734	1.0000	0.5590	0.0114
25	1.2919	1.0000	0.4796	0.0000
26	1.3333	1.0000	0.5251	0.0040
27	1.2975	1.0000	0.4939	0.0037
28	1.3407	1.0000	0.5422	-0.0000
29	1.3021	1.0000	0.5525	0.0073
30	1.3448	1.0000	0.6161	0.0041
31	1.5516	1.0000	0.6262	0.0000

Fig. 8. Exemple of the α-stable estimated parameters fitted to the residuals of the linear model (2).

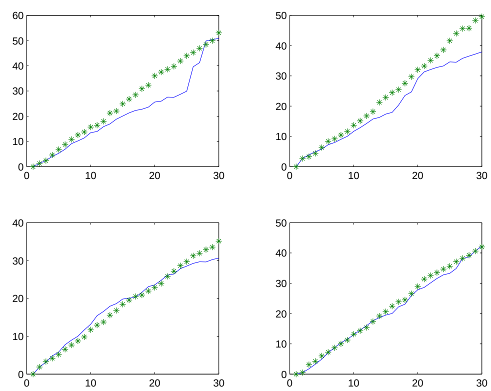

Fig. 9. Exemples of real delays(plotted with stars) and the corresponding predicted delays using the linear model (2)

(2). Also a spearman test reveals the linear model residuals $(\epsilon_1, ..., \epsilon_{N-1})$ are independents.

V. Theoretical tail probability of the delays between arrivals

The aim of this section is to establish the theoretical probability distribution of the delays between arrivals in the impulse response of the 60Ghz channel and hence justify the results observed in the experimental data. Each received ray has passed through a countable number of obstacles encountered in the transmission environment. Each obstacle reflects it and redirects the ray to another obstacle or eventually to the receiver. Note that many rays may arrive at the same time but this fact will only affect the amplitude of the signal and not the time of arrivals.

To formulate mathematically our problem, we define : N, the number of obstacles that a ray has effectively encountered before arriving to the receiving antenna (N can tend towards infinity), $d_{k,i}$, the duration between the reflection on the k^{th} and the $(k+1)^{th}$ reflectors for a given path i. T, the total transmission duration : $T = \sum_{k=1}^{N} d_{k,i}$. If we denote by t_0 the deterministic value of the first path time of arrival, the i^{th} ray observed in the impulse response arrives with the delay :

$$\tau_i = \sum_{k=1}^{N_i} d_{k,i} - t_0. \tag{3}$$

Since the number N of reflectors encountered by a received ray depends on the random structure and geometry of the room, it is an integer random variable. Similarly the time to pass from an obstacle to another one depends on the random distribution and positions of the obstacles existing in the environment, hence we will suppose that the d_k are mutually independents and identically distributed random variables. In addition we suppose that they are independent of N.

A basic assumption in our model, as well as many of the previous models of the impulse response channel [13], is that the number of obstacles (that a received ray has effectively encountered in his path the receiver) is a Poisson point process with parameter increasing with the order of arrival[2]. This assumption is inspired from the fact that : in the case of static environment, which was the case of measurements rooms : the long time takes an arrived ray to reach the receiver, the more likely it has followed a complicated path and hence has encountered many obstacles. We formulate this assumption by :

H1) : the number of encountered obstacles, by the i^{th} arrived ray, is a Poisson random variable with parameter δ_i^{α} where $(0 < \alpha < 2)$ and δ_i is positive number that increases to infinity as the order of arrival i increases : more precisely δ_i^{α} is the mean number of obstacles encountered by the i^{th} arrived path of duration δ_i. The free parameter α controls how the mean number of obstacles tends toward infinity as time passes. It can be linked to the characteristics of the transmission environment (for instance the disposition of the obstacles in the room...etc).

In reality, when the number of encountered obstacles is very large (this means that the ray follows a very complicated path to arrive), the power of the received signal will be very small, consequently the passage from one obstacle to another becomes increasingly difficult. In other words, the probability to encounter a next obstacle will decrease with the increase of its distance from its precedent. It will also decrease with the growth of the time passed since the first emission of the ray. For convenience,[3] we suppose the next hypothesis on the probability distribution of $d_{k,i}$:

H2) : Suppose that the probability distribution of the delay $d_{k,i}$ to pass from an obstacle to another one is given by :

$$P(d_{k,i} > x) = \begin{cases} \delta_i^{-\alpha} x^{-\alpha} & if \quad x > \frac{1}{\delta_i} \\ 1 & if \quad x \leq \frac{1}{\delta_i}. \end{cases}$$

[2] Arrivals are ordered in time so saying that the order of arrival increases means that the time passes.

[3] Note that we haven't verified, physically or by experience, this two hypothesis.

A. Proposition

Under the hypothesis (H_1) and (H_2) the delay time given by (3) is a random variable verifying :

- if $0 < \alpha < 1$ the random variable τ_i converges in distribution, as δ_i goes to infinity, to an $\alpha-$stable distribution totaly skewed to the right with a scale parameter given by the formula $\sigma^\alpha = \Gamma(1-\alpha)\cos(\frac{\pi\alpha}{2}) > 0$.
- if $1 < \alpha < 2$ then the random variable $\tau_i - \frac{\alpha}{\alpha-1}[\frac{1}{\delta_i}]^{1-\alpha}$ converges in distribution, as δ_i goes to infinity, to an $\alpha-$stable distribution totaly skewed to the right with a scale parameter verifying $\sigma^\alpha = \Gamma(1-\alpha)\cos(\frac{\pi\alpha}{2}) > 0$.

where Γ is the usual gamma function.
Proof is given in appendix.

B. Remarks

This proposition gives a theoretical justification for modelling the delays, in the impulse response of the 60GHz channel, by α-stable distributions, and also confirms the empirical results, and curve fitting of their distributions. As we noticed when fitting the theoretical distribution to the measured data, accuracy increases with the path order of arrival. This fact can occur because the complication of the paths let δ goes to zero and hence the sum (3) becomes stable due to this last proposition. It also explain the right dissymmetry of the observations about their means. The estimation of the index of variability gives values between one and two which let $\delta^{1-\alpha}$ increases slowly to infinity as delta goes to 0 which is the case in our observed means.

VI. Conclusion

Channel paths' arrival times play an important role on the behavior of a communication system. An accurate model is then necessary for simulation and development of communication chains. Previous models, based on Poisson processes are difficult to generalize to new sets of measurements. We have proposed in this paper a new approach with α-stable distributions. In contrast of the Poisson distribution they have four free parameters that can be adjusted to provide a good fit. On the other hand they can be seen as a mixture of Poisson processes which leads to a theoretical justification of their use. Besides, we have shown a good fit between measurements and those distributions and efficient methods exist for the calculation of the parameters. Those distributions appear to be a good solution for modelling the paths' arrival times for the 60GHz channel and could certainly be extended to new configurations and frequency bands.

Appendix

Proof: For simplicity and without loss of generality , we omit the index i and we take $\delta = \frac{1}{\delta_i}$. We will show the proposition (V-A) when $1 < \alpha < 2$. The other case $(0 < \alpha < 1)$ is demonstrated in [15]. For this purpose it suffices to show that the characteristic function of $Y_\delta = \tau - \frac{\alpha}{\alpha-1}\delta^{1-\alpha}$ converges to the characteristic function of an $\alpha-$stable distribution. By using the condition of independence we have :

$$\begin{aligned}\Phi_\tau^\delta(\theta) &= E\left[e^{i\theta Y_\delta}\right]\\ &= E\left[\exp(i\theta\sum_{k=1}^{N} d_k - \frac{\alpha}{\alpha-1}\delta^{1-\alpha})\right]\\ &= e^{i\frac{\alpha}{\alpha-1}\theta\delta^{1-\alpha}}E\left[\left[Ee^{i\theta d_1}\right]^N\right].\end{aligned}$$

Since N is a poisson random variable with parameter $\delta^{-\alpha}$ so the expectation in the second hand side of the last equality is obtained through it's factorial generating function, hence :

$$E\left[Ee^{i\theta d_1}\right]^N = \exp[\delta^{-\alpha}(Ee^{i\theta d_1}-1)]$$

so,

$$\begin{aligned}\Phi_\tau^\delta(\theta) &= \exp\left[\delta^{-\alpha}(Ee^{i\theta d_1}-1) - i\theta\frac{\alpha}{\alpha-1}\delta^{1-\alpha}\right]\\ &= \exp[\delta^{-\alpha}\int_{-\infty}^{\infty}(e^{i\theta\zeta}-1)dF_\delta(\zeta) - i\theta\frac{\alpha}{\alpha-1}\delta^{1-\alpha}]\\ &= \exp[\int_\delta^\infty \alpha\frac{e^{i\theta\zeta}-1}{\zeta^{\alpha+1}}d\zeta - i\theta\frac{\alpha}{\alpha-1}\delta^{1-\alpha}]\\ &= \exp[\alpha\int_\delta^\infty \frac{e^{i\theta\zeta}-1-i\theta\zeta}{\zeta^{\alpha+1}}d\zeta].\end{aligned}$$

As δ converges to zero, then for $\theta \geq 0$ the characteristic function is equal :

$$\Phi_\tau(\theta) = \exp[\alpha\int_0^\infty \frac{e^{i\theta\zeta}-1-i\theta\zeta}{\zeta^{\alpha+1}}d\zeta]$$

by a simple integration by parts and the fact that $1 < \alpha < 2$ we have :

$$\alpha\int_0^\infty \frac{e^{i\theta\zeta}-1-i\theta\zeta}{\zeta^{\alpha+1}}d\zeta = i\theta\int_0^\infty \frac{e^{i\theta\zeta}-1}{\zeta^{\alpha}}d\zeta$$

the integral in the second hand side of the last equality is obtained through the characteristic function of the gamma distribution calculated in Feller [16] that is for every $0 < \alpha < 1$ and $\theta \geq 0$ we have :

$$\int_0^\infty \frac{e^{i\theta\zeta}-1}{\zeta^{\alpha+1}}d\zeta = -\theta^\alpha\frac{\Gamma(1-\alpha)}{\alpha}e^{-i\frac{\pi\alpha}{2}}$$

which implies that in the case of $1 < \alpha < 2$ and $\theta \geq 0$ we have:

$$\begin{aligned}\alpha\int_0^\infty \frac{e^{i\theta\zeta}-1-i\theta\zeta}{\zeta^{\alpha+1}}d\zeta &= -i\theta^\alpha\frac{\Gamma(2-\alpha)}{(\alpha-1)}e^{-i\pi\frac{\alpha-1}{2}}\\ &= -|\theta|^\alpha\frac{\Gamma(2-\alpha)}{(1-\alpha)}e^{-i\frac{\pi\alpha}{2}}\\ &= -|\theta|^\alpha\Gamma(1-\alpha)e^{-i\frac{\pi\alpha}{2}}\end{aligned}$$

If θ is negative we use the fact that this last integral is equal to the conjugate of the second hand side of the last equality, which leads to :

$$\Phi_\tau(\theta) = \exp[-|\theta|^\alpha\cos(\frac{\pi\alpha}{2})\Gamma(1-\alpha)[1-\mathrm{i}\ \mathrm{sign}(\theta)\tan(\frac{\pi\alpha}{2})]$$

which is nothing but the characteristic function of an α-stable distribution totally skewed to the right with scale parameter verifying $\sigma^\alpha = \Gamma(1-\alpha)\cos(\frac{\pi\alpha}{2}) > 0$. ∎

REFERENCES

[1] Smulders P. (2002), "*Exploiting the 60 GHz band for local wireless multimedia access : prospects and future direction*", IEEE Communications Magazine, January 2002, pp 140-147.

[2] Turin G. L. , Calpp F. D. ,Johnston T. L. ,Fine S. B. andLavry D. , (1972), "*A statistical model of urban multipath propagation*", IEEE, trans., Veh. Tech., vol. VT-21, pp 1-9.

[3] Suziki H. (1977) "*A statistical model for urban radio propagation*", IEEE Trans., commun., vol. COM-25, pp 673-680.

[4] Saleh A.A.M , Valenzuela R.A. , *A statistical model for indoor multipath propagation*, IEEE Journal on selected areas of communication, Vol. SAC-5, 128-137, February 1987.

[5] Clavier L. ,Rachdi M., Fryziel M. , Delignon Y. , LeTuc V., Garnier C. , Rolland P.A., (2001) " *Wide band 60GHz indoor channel : Characterization and statistical modelling*" IEEE 54th VTC fall, Atlantic City; NJ USA, 7-11 October, 2001.

[6] Hashemi H. (1979) "*Simulation of the urban propagation channel*". IEEE Transactions on Vehicular Technology , vol. 28, no. 3, pp. 213-225, August 1979.

[7] Michel FRYZIEL, Christophe LOYEZ, Laurent CLAVIER, Nathalie ROLLAND, Paul Alain ROLLAND, "*Path loss model of the 60 GHz indoor radio channel*", Microwave and optical technology letters, 2002.

[8] Dumouchel W.H., (1983) "*estimating the stable index α in order to measure tail thickness : a critique*" Amer.statist., vol. 11, N.4,pp, 1019-1031.

[9] Zolotarev V. M., (1986) "*one dimensional stable distributions*", Providence, RI, American Mathematical Society.

[10] McCulloch J. H. , (1986), "*simple consistent estimators of stable distribution parameters*, Comm.Statist.Simula., Vol. 14, N 4, pp 1109-1136.

[11] Famma E.F. and Roll, (1971) "*parameter estimates for symmetric stable distributions*", J.Amer.statist.assoc., vol. 66 (June), pp. 331-338.

[12] Koutrouvelis I.A. , (1981) "*An iterative procedure for the estimation of the parameters of stable laws*", Commun.Statist.Simula, Vol. 10, N 1, pp17-28.

[13] Nikias C. L. , Shao M. (1996), "*Signal processing with alpha-stable distributions and applications*" J.Wiley, Wiley inter science.

[14] Weron R. Wroclow, "*Performance of the estimators of stable law parameters*", AMS 1991 subject classifications.primary 60E07,62G07.

[15] Samorodnitsky G. and Taqqu M.S. , (1994) "*Stable non Gaussian random processes : stochastic model with infinite variance*", New York, Chapman & Hall.

[16] Feller W. (1971) "*an introduction to probability theory and it's applications*", Vol 2, New York, J.Wiley.