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Document de travail

[2011-1]

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Université Lille Nord de France
Pôle de Recherche et d'Enseignement Supérieur

Université Lille 1 Sciences et Technologies
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Education, Wage Inequality and Growth☆

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Abstract

We model a successive-generation economy in which parents, motivated by family altruism, decide to finance or not their offspring's capital accumulation on the basis of their altruistic motive, their own income and the equilibrium ratio between skilled-labor and unskilled-labor wages. The question we ask is how the growth process in this economy shapes the wage inequality and the split of the population in two classes. We study the transitional dynamics of human capital accumulation and of wage inequality. First, we prove the existence of equilibrium paths. Then we show that there exists a continuum of steady-state equilibria and prove the convergence of each equilibrium path to one of the steady-state equilibrium. Also we look at the relationship between inequality and output on the set of steady states and find that this relationship is ambiguous. Finally, we deal with an endogenous-growth version of our model, which displays the ambiguous relationship between inequality and the rate of growth.

Keywords: Growth; Inequality; Human capital

JEL classification: O41; O15; D31; D91

☆The research developed in this paper is part of the project *Mondialisation, restructurations et déclassement social et professionnel* (MONDES), which is financed by the French National Agency of Research (ANR) through its programme *Vulnérabilités : à l'articulation du sanitaire et du social*. We thank the ANR for its financial support. The usual disclaimer applies.

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1. Introduction

The understanding of the role of human capital in economic activity has been decisively spurred by the work of G. Becker (1964). Human capital is a major feature of economic relations inside the family and, as such, is one of the key concepts for understanding of the individual decision-making over the life-cycle and the functioning of labor markets. On their side macroeconomists, and especially economic growth theorists, investigated the role of human capital in determining the growth rate of economies in the short and the long run[1].

Modern growth theory has long been relying on the fiction of the representative agent. However, societies are patently not homogeneous, whether in incomes, wealth, or many other dimensions. In a sense, the question of inequality and its link with the growth process is an old one. The classical argument is that inequality is good for growth because the wealthy are more patient and accumulate more assets than the poor. Over the last 20 or 30 years, the nexus between inequality and growth has attracted a great deal of interest. We can distinguish two types of questions about this nexus. The first one is: “How does inequality affect growth?”, namely do unequal economies perform better than those more equal? The policy implications of this first type of question are relevant for policies aiming at redistributing income and wealth among households. The role played by capital market imperfection in discouraging human capital accumulation has been stressed by several important contributions (see, e.g., Loury (1981), Galor and Zeira (1993))[2].

The second type of question about the inequality-growth nexus is then the following: “How does the growth process affect in turn inequality?”, namely

[1] To mention but one major contribution, see, *e.g.*, Lucas (1988).

[2] Other arguments have been put forward to emphasize the negative relationship between inequality and growth: political economy arguments (Alesina and Rodrik (1994), Bertola (1993), Persson and Tabellini (1994)), or social conflict arguments (Alesina and Perotti (1996) and Borissov and Lambrecht (2009)).

what is the feedback of growth into the evolution of inequality across time. This second question is relevant also for the first one. Indeed, as Aghion et al. (1999) argue, if redistribution creates a virtuous circle by alleviating credit constraints to human capital accumulation, these policy efforts might be vain if growth in turn worsen inequalities. A virtuous circle would be more or less offset by a vicious circle. This paper is mainly about the second type of question.

It deals with it by introducing heterogeneity coming from the functioning of the labor markets, namely the occupational heterogeneity. According to this heterogneneity, workers in the economy need to occupy positions of different skill levels and get different endogenously determined wages. As a consequence, educational decisions determine not only the individual stock of human capital but also influences the choice of occupation and the wages structure.

Ray (2006) examines equilibrium paths of an economy in which skilled and unskilled labor are necessary to produce. Each generation decides whether to finance the offspring's acquisition of human capital out of a dynastic (Barro, 1974) altruistic motive to finance educational expenses. Since both skilled and unskilled categories of labor are necessary to production, equilibrium wages adjust to insure that each category of labor has positive supply, i.e. that one share of the population occupies low-skill jobs and the other high-skill jobs. As a result, inequality inside each generation must emerge. Ray (2006) shows that this intragenerational inequality is persistent in the long run. Moreover a continuum of steady states is possible.

In a recent contribution, Mookherjee and Ray (forthcoming) extend and modify Ray's (2006) model. They provide a small-open economy model with physical capital, a continuum of occupations, training costs and a mix of utility-based and wealth-based motivation for bequest. They derive conditions under which the steady state exhibits inequality. These conditions rely on the share of occupations with high training costs[3] being non degenerate.

Our article is close to Ray's (2006) and Mookherjee and Ray's (forthcom-

[3]Higher than an endogenous threshold.

ing) articles in the sense that it looks at the persistence of inequality in an economy populated with altruists facing occupational heterogeneity. Its main contribution is that it explicitly deploys and analyses the dynamics of human capital accumulation, and hence allows for different levels of human capital for the skilled workers.

We assume that individuals care about their offspring's net disposable income and that there is an accumulative education function. Becker and Tomes (1979) used this set of hypotheses to analyse the equilibrium distribution of income and intergenerational mobility. They labeled the approach based on the offspring's wealth by the term "quality of the children". They claimed that the implications in terms of income distribution of this approach are similar to those of Barro's (1974) "dynastic altruism" approach, in which altruists care about their offsring's utility. Lambrecht et al (2005) and Lambrecht et al (2006) studied the properties of fiscal policies under this approach, which they label "family altruism". They find less clear cut conclusions: pay-as-you-go policies are neutral but public debt is not. The family altruism approach enables to study the transitional dynamics of physical and/or human capital[4]. As an altruistic bequest motive, it is also preferable to Andreoni's (1989) joy-of-giving or warm-glow motive because the latter is insensitive to the economic situation of the beneficiaries of transfers.

To summarize, this paper is based on the threefold assumption of (i) family altruism (ii) accumulative human capital and (iii) the existence of two distinct occupational choices (high-skil and low-skill jobs). Moreover, in the high-skill occupations, there is room for heterogeneity in human capital, and hence in income. In that sense it combines (i) the neoclassical approach which sees human capital as efficiency units of labor whose individual endowments vary across the skilled workers and (ii) the approach which emphasizes the role of indivisibilities in occupational choice[5].

[4] We confine our analysis to human capital only.

[5] Indivisibilities may also come from the educational system like in Chusseau and Hellier (2010).

We study the equilibrium paths along which human capital is accumulated differently across each generation's family. The main results are the following. First, we prove that there exists a unique intertemporal equilibrium path starting from any initial distribution of human capital. Secondly, we establish necessary and sufficient conditions for the existence of stationary equilibrium paths compatible with inequality in income among families and show that there exist a multiplicity of steady states. At these steady states, one share of the population permanently supply unskilled labor while the remaining share maintain a unique and constant human capital across generations and supplies skilled labor. Then we show that any equilibrium path converges to a steady state equilibrium with inequality. Finally, we propose an endogenous version of the model by assuming that the productivity of unskilled labor benefits from the accumulation of human capital. In this version of the model, it is shown that the relationship between inequality and growth is ambiguous.

The paper starts with the presentation of the model in Section 2. Then the competitive equilibrium is analyzed in Section 3. Section 4 studies steady state equilibria and Section 5 establishes the convergence to equilibrium paths to steady state equilibrium. In Section 6 we shortly describe an endogenous-growth version of our model. Section 7 concludes.

2. The model

2.1. The firms

At each time t, the output of the representative firm, Y_t, is determined by the Cobb-Douglas production function:

$$Y_t = (H_t)^\alpha (L_t)^{1-\alpha},$$

where H_t is the supply of human capital and L_t is the supply of unskilled labor. The wage rates of unskilled labor and human capital, w_t^L and w_t^H, are determined by their marginal products:

$$w_t^H = \alpha \left(\frac{L_t}{H_t}\right)^{1-\alpha}, \; w_t^L = (1-\alpha)\left(\frac{H_t}{L_t}\right)^{\alpha}.$$

Output is is either consumed or spent on education.

2.2. The households

Our model is one of successive generations. Each agent lives for one time period and has one offspring. The set of dynasties is the interval $[0,1]$. Each dynasty is denoted by the variable i. Each agent is endowed with one unit of unskilled labor force that requires no higher education. Some agents (not necessarily all) are also endowed with some amount of human capital. We will call agents with positive endowment of human capital educated agents and those with zero endowment of human capital uneducated agents.

The human capital of the agent of dynasty i living at time t, $h_t(i)$, depends on the human capital of his parent, $h_{t-1}(i)$ and the amount of money the parent spent for his higher education, $e_{t-1}(i)$. We assume that this dependence is as follows:

$$h_t(i) = e_{t-1}(i)^{\kappa}(h_{t-1}(i)+1)^{1-\kappa}, \quad 0<\kappa<1, \quad \forall t \geq 0. \tag{1}$$

It follows that if the parent of an individual spent nothing on his education, the human capital of this individual is nil.

Consider the individual that belongs to dynasty i and lives in period t. During this period, he supplies inelastically either unskilled labor or human capital. If $w_t^L > w_t^H h_t(i)$, he supply one unit of unskilled labor. If $w_t^L < w_t^H h_t(i)$, he supplies $h_t(i)$ units of human capital. If $w_t^L = w_t^H h_t(i)$, he is indifferent in this respect. Thus, his total income is

$$\omega_t(i) = \max\{w_t^L, w_t^H h_t(i)\}, \quad \forall t \geq 0.$$

It is divides between consumption $c_t(i)$ and education expenditure for his offspring $e_t(i)$. This education expenditure is motivated by family altruism (see Lambrecht *et al.* 2005 and Lambrecht *et al.* 2006), i.e. by the concern for the offspring's total income $\omega_{t+1}(i) = \max\{w_{t+1}^L, w_{t+1}^H h_{t+1}(i)\}$, where w_{t+1}^L and w_{t+1}^H are the expected time $t+1$ wage rates and $h_{t+1}(i)$ is the offspring's human capital. According to (1) at time $t+1$, spending $e_t(i)$ on the offspring's education determines the latter's human capital, and thus his total income.

The individuals' preferences are defined over consumption $c_t(i)$ and the offspring's expected total income $\omega_{t+1}(i) = \max\{w^L_{t+1}, w^H_{t+1}h_t(i)\}$ They are represented by the following log-linear utility function: $\ln c_t(i) + \ln \omega_{t+1}(i)$. Hence the individual maximizes his utility function under his budget constraints considering current wages and expectations on next period wages as given. We state this problem as follows

$$\max_{c_t(i)\geq 0, e_t(i)\geq 0} \ln c_t(i) + \ln \omega_{t+1}(i)$$

under the following constraints:

$$\begin{aligned} e_t(i) + c_t(i) &= \omega_t(i), \\ \omega_{t+1}(i) &= \max\{w^L_{t+1}, w^H_{t+1}h_{t+1}(i)\}, \\ h_{t+1}(i) &= e_t(i)^\kappa (h_t(i)+1)^{1-\kappa}. \end{aligned}$$

This problem can be rewritten as follows:

$$\max_{0\leq e_t(i)\leq \omega_t(i)} \ln(\omega_t(i) - e_t(i)) + \ln(\max\{w^L_{t+1}, w^H_{t+1}e_t(i)^\kappa (h_t(i)+1)^{1-\kappa}\}). \quad (2)$$

Problem (2) can be solved in two steps. First we solve the two sub-problems defined by the two alternatives of the max function,

$$\max_{0\leq e_t(i)\leq \omega_t(i)} \ln(\omega_t(i) - e_t(i)) + \ln(w^L_{t+1}), \quad (3)$$

and

$$\max_{0\leq e_t(i)\leq \omega_t(i)} \ln(\omega_t(i) - e_t(i)) + \ln(w^H_{t+1}e_t(i)^\kappa (h_t(i)+1)^{1-\kappa}), \quad (4)$$

and then we select the solution leading to the highest utility.

The solution to problem (3) is $e_t(i) = 0$. If at time $t+1$ dynasty i is going to supply unskilled labor on the labor market, then at time t its expenditure on education is nil and all income $\omega_t(i)$ is spent on consumption. Hence the human capital of this dynasty at time $t+1$, $h_{t+1}(i)$, is also nil. The value of problem (3) is

$$V^L\big(\omega_t(i), h_t(i)\big) := \ln \omega_t(i) + \ln w^L_{t+1}.$$

The solution to problem (4) is

$$e_t(i) = \hat{e}^H\big(\omega_t(i), h_t(i)\big) := \frac{\kappa}{1+\kappa}\omega_t(i).$$

If at time $t+1$ dynasty i is going to supply human capital on the labor market, then at time t its expenditure on education is equal to the fraction $\frac{\kappa}{1+\kappa}$ of its income $\omega_t(i)$. The rest of the income, $\frac{1}{1+\kappa}\omega_t(i)$, is spent on consumption. Thus, the endowment of human capital of agent that belongs to dynast i and lives at time $t+1$ is equal to

$$\hat{h}^H\big(\omega_t(i), h_t(i)\big) := \gamma\big(\omega_t(i)\big)^{\kappa}\big(1+h_t(i)\big)^{1-\kappa}, \tag{5}$$

where

$$\gamma = \left(\frac{\kappa}{1+\kappa}\right)^{\kappa}.$$

The value of problem (4) is

$$V^H\big(\omega_t(i), h_t(i)\big) := \ln\omega_t(i) - \ln(1+\kappa) + \ln w^H_{t+1} + \ln\hat{h}^H\big(\omega_t(i), h_t(i)\big).$$

It is clear that if $V^H\big(\omega_t(i), h_t(i)\big) < V^L\big(\omega_t(i), h_t(i)\big)$, then the unique solution to problem (2) coincides with the solution to problem (3). If $V^H\big(\omega_t(i), h_t(i)\big) > V^L\big(\omega_t(i), h_t(i)\big)$, then the unique solution to problem (2) coincides with the solution to problem (4), and if $V^H\big(\omega_t(i), h_t(i)\big) = V^L\big(\omega_t(i), h_t(i)\big)$, then the solutions of both (3) and (2) are solutions to (2). It is easily checked that

$$V^H\big(\omega_t(i), h_t(i)\big) \gtreqless V^L\big(\omega_t(i), h_t(i)\big) \;\Leftrightarrow\; \frac{\hat{h}^H\big(\omega_t(i), h_t(i)\big)}{1+\kappa} \gtreqless \frac{w^L_{t+1}}{w^H_{t+1}}.$$

Thus we can formulate the following proposition.

Proposition 1. 1) *If*

$$\frac{\hat{h}^H\big(\omega_t(i), h_t(i)\big)}{1+\kappa} > \frac{w^L_{t+1}}{w^H_{t+1}}, \tag{6}$$

then $e_t(i) = \hat{e}^H\big(\omega_t(i), h_t(i)\big)$ *is the unique solution to (2);*

2) *if*

$$\frac{\hat{h}^H\big(\omega_t(i), h_t(i)\big)}{1+\kappa} < \frac{w^L_{t+1}}{w^H_{t+1}}, \tag{7}$$

then $e_t(i) = 0$ *is the unique solution to (2);*

3) *if*

$$\frac{\hat{h}^H\big(\omega_t(i), h_t(i)\big)}{1+\kappa} = \frac{w_{t+1}^L}{w_{t+1}^H}, \tag{8}$$

then there are two solutions to (2): $e_t(i) = 0$ *and* $e_t(i) = \hat{e}^H\big(\omega_t(i), h_t(i)\big)$.

At each time t it is convenient to order the set of dynasties in a way such that the function $h_t(\cdot)$ defined on the interval $[0,1]$ and describing the distribution of human capital across dynasties is non-decreasing. At the same time it follows from (5) that if for some i and j, $h_t(i) \geq h_t(j)$ and $\omega_t(i) \geq \omega_t(j)$, then $\hat{h}^H\big(\omega_t(i), h_t(i)\big) \geq \hat{h}^H\big(\omega_t(j), h_t(j)\big)$. This give us the opportunity to order the set of dynasties at time 0 and restrict our consideration to paths of the economy such that all functions $h_t(\cdot)$ are non-decreasing at the initial order.

3. Competitive equilibrium

To study the general equilibrium of this economy we proceed in two steps[6]. We first study the time t temporary equilibrium in subsection 3.1 given past variables and expectations of the future. In subsection 3.2, we then describe the intertemporal equilibrium with perfect foresight as a sequence of temporary equilibria with some adequate initial conditions and rule for expectations.

3.1. Time t temporary equilibrium

To define the time t temporary equilibrium, we consider all past variables and expectations of the future as given. The latter are expectations of the next period wages w_{t+1}^L and w_{t+1}^H and the former are the time $t-1$ human capital levels, $h_{t-1}(i)$ $\forall i \in [0,1]$, total incomes $\omega_{t-1}(i) = \max\{w_{t-1}^L, w_{t-1}^H h_{t-1}(i)\}$ $\forall i \in [0,1]$, and educational spendings $e_{t-1}(i)$ $\forall i \in [0,1]$. Since these given past variables determine time t human capital levels $h_t(i)$ $\forall i \in [0,1]$, we can say that these levels are completely pre-determined by time $t-1$ decisions. To be more

[6]See Hicks (1939) or, more recently Grandmont (1983) for the articulation of these two steps.

precise, all we need to know to construct the time t temporary equilibrium is the function $h_t(\cdot)$.

Let us assume that the function $h_t(\cdot)$ is non-decreasing and upper semi-continuous and that $\int_0^1 h_t(i)di > 0$.

A time t *temporary equilibrium* is defined by a quadruple of functions $\{\omega_t(\cdot), c_t(\cdot), e_t(\cdot), h_{t+1}(\cdot)\}$, defined on $[0,1]$, a pair of prices $\{w_t^L, w_t^H\}$, a triplet of aggregate variables $\{L_t, H_t, Y_t\}$ and a *pivotal* dynasty i_t^H satisfying the following requirements:

- all agents, households and firms, are at their optima;
- the set of dynasties supplying unskilled labor at time t is $[0, i_t^H) = \{i \mid 0 \le i < i_t^H\}$ and the set of dynasties supplying human capital is $[i_t^H, 1] = \{i \mid i_t^H \le i \le 1\}$;
- all markets clear.

It should be noticed that the pivotal dynasty i_t^H shows the fraction of unskilled labor suppliers in the population. The fraction of human capital suppliers is respectively $1 - i_t^H$.

To make our presentation simple we also impose the following requirement on temporary equilibrium, which will not lead to any loss of generality:

- $h_{t+1}(i)$ is a non-decreasing upper semi-continuous function defined on $[0,1]$.

We determine this equilibrium at time t by writing the variables of the above-mentioned tuples as functions of past variables and expectations. To find a temporary equilibrium at time t it is sufficient to determine the pivotal dynasty i_t^H. Knowing it, one can easily determine the equilibrium values of all other variables.

In equilibrium, the supply of unskilled labor is equal to

$$L_t = \int_0^{i_t^H} di = i_t^H, \tag{9}$$

the supply of human capital is equal to

$$H_t = \int_{i_t^H}^{1} h_t(i)di \tag{10}$$

and human capital and unskilled labor are paid at their marginal products:

$$w_t^H = \alpha \left(\frac{i_t^H}{\int_{i_t^H}^1 h_t(i)di} \right)^{1-\alpha}, \; w_t^L = (1-\alpha) \left(\frac{\int_{i_t^H}^1 h_t(i)di}{i_t^H} \right)^{\alpha}. \tag{11}$$

Therefore,

$$\frac{w_t^L}{w_t^H} = \frac{(1-\alpha)}{\alpha} \left(\frac{\int_{i_t^H}^1 h_t(i)di}{i_t^H} \right).$$

Also, in equilibrium, we need to have

$$w_t^H h_t(i) \leq w_t^L \;\Leftrightarrow\; h_t(i) \leq \frac{w_t^L}{w_t^H}, \; 0 \leq i < i_t^H,$$

and

$$w_t^H h_t(i) \geq w_t^L \;\Leftrightarrow\; h_t(i) \geq \frac{w_t^L}{w_t^H}, \; i_t^H \leq i \leq 1.$$

Therefore, given the time t human capital levels $h_t(i) \forall \; i \in [0,1]$, the time t equilibrium pivotal dynasty, i_t^H, is determined by the following conditions:

$$h_t(i) \leq \frac{(1-\alpha)}{\alpha} \left(\frac{\int_{i_t^H}^1 h_t(i)di}{i_t^H} \right), \; 0 \leq i < i_t^H, \tag{12}$$

$$h_t(i) \geq \frac{(1-\alpha)}{\alpha} \left(\frac{\int_{i_t^H}^1 h_t(i)di}{i_t^H} \right), \; i_t^H \leq i \leq 1. \tag{13}$$

It is clear that $\frac{(1-\alpha)}{\alpha} \left(\frac{\int_{i^H}^1 h_t(i)di}{i^H} \right)$ is a continuous decreasing function of i^H. At the same time $h_t(i)$ is a non-decreasing function of i. Therefore, to find the pivotal dynasty it is sufficient to "solve" the following equation in i^H:

$$h_t(i^H) = \frac{(1-\alpha)}{\alpha} \left(\frac{\int_{i^H}^1 h_t(i)di}{i^H} \right). \tag{14}$$

If the solution to equation (14) exists, the time t equilibrium pivotal dynasty coincides with this solution. In this case this dynasty is the one with human capital just equal to the ratio between unskilled labor wage rate and human capital wage rate and hence for this dynasty supplying unskilled labor will result in the same income as supplying human capital:

$$w_t^L = w_t^H h_t(i_t^H).$$

A solution to (14) may not exist because $h_t(i)$ is not necessarily continuous. But even in the case of non-existence there is a unique i_t^H satisfying (12)-(13).

Knowing i_t^H, we get H_t, L_t, w_t^H and w_t^L from (9)–(11). Also we are able to determine the total income of all households:

$$\omega_t(i) = \begin{cases} w_t^L, & 0 \le i < i_t^H, \\ w_t^H h_t(i), & i_t^H \le i \le 1. \end{cases}$$

With the pairs $\{h_t(i), \omega_t(i)\}$ $\forall i$, we can now determine the time t equilibrium educational expenditures, $e_t(i)$, i.e. the optimal educational expenditures at equilibrium prices given expectations on next period wage rates w_{t+1}^L and w_{t+1}^H. Once this variable is determined, it will give us the next period distribution of human capital, the $h_{t+1}(i)$'s. Here we should notice that the functions $e_t(\cdot)$ and $h_{t+1}(\cdot)$ are not necessarily uniquely determined, because for i satisfying (8), $e_t(i)$ is equal to either 0 or $\hat{e}^H\big(\omega_t(i), h_t(i)\big)$. However, this non-uniqueness plays no role in our model, because, as will be shown in the next subsection, in intertemporal equilibrium non-uniqueness does not appear.

For short, in what follows we identify any temporary equilibrium at time t with the couple $\{i_t^H, h_{t+1}(\cdot)\}$.

3.2. The intertemporal equilibrium with perfect foresight

Suppose we are given an initial state of the economy represented by a non-decreasing upper semi-continuous function $h_0(\cdot)$ showing the distribution of human capital across dynasties at the initial time. We assume that $\int_0^1 h_0(i)di > 0$ and define an *intertemporal equilibrium* path $\{i_t^H, h_{t+1}(\cdot)\}_{t=0}^{\infty}$ starting from $h_0(\cdot)$ as a sequence of temporary equilibria, such that at each time t each dynasty has perfect foresight, that is, correctly anticipate time $t+1$ wage rates.

Theorem 1. *For any initial state $h_0(\cdot)$ there is a unique intertemporal equilibrium path starting from this initial state.*

Proof. Let $h_t(\cdot)$ be given. Let further i_t^H, w_t^L, w_t^H and $\omega_t(\cdot)$ be found as described in the previous section. To find the function $h_{t+1}(\cdot)$, we start with using the time $t+1$ human capital function associated with positive investment in education given by equation (5) in section 2.2, namely $\hat{h}^H\big(\omega_t(i), h_t(i)\big)$. Since the analysis will focus on the pivotal dynasty and all the arguments of this function depend on i, we re-write it as a function $\tilde{h}_{t+1}$ of i.

Namely, let the function $\tilde{h}_{t+1} : [0,1] \to \mathbb{R}_+$ be defined by

$$\tilde{h}_{t+1}(i) = \hat{h}^H\big(\omega_t(i), h_t(i)\big) \ (= \gamma\,[\omega_t(i)]^\kappa \,\big(1 + h_t(i)\big)^{1-\kappa}).$$

Clearly, $\tilde{h}_{t+1}(\cdot)$ is a non-decreasing upper semi-continuous function such that

$$\tilde{h}_{t+1}(i) = \gamma \left[w_t^L\right]^\kappa, \ 0 \leq i < i_t^H, \tag{15}$$

$$\tilde{h}_{t+1}(i) = \gamma \left[w_t^H\right]^\kappa \psi(h_t(i)), \ i_t^H \leq i \leq 1, \tag{16}$$

where the function $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is given by

$$\psi(h) = h^\kappa (h+1)^{1-\kappa}.$$

Let further the function $\mathcal{H}_{t+1} : [0,1] \to \mathbb{R}_+$ be defined by

$$\mathcal{H}_{t+1}(i^H) = \int_{i^H}^1 \tilde{h}_{t+1}(i)di. \tag{17}$$

This function is continuous and decreasing. It shows the dependence of the aggregate supply of human capital on the pivotal dynasty. In equilibrium at time $t+1$ the ratio of the wage rates of common labor and human capital is endogenously determined by the marginal productivities of these inputs, which in turn is determined by the relative masses of these inputs. So we have:

$$\frac{1-\alpha}{\alpha}\frac{\mathcal{H}_{t+1}(i_{t+1}^H)}{i_{t+1}^H} = \frac{w_{t+1}^L}{w_{t+1}^H}$$

and, at the same time, from Proposition 1 and the definition of the function $\tilde{h}_{t+1}(\cdot)$

$$\frac{\tilde{h}_{t+1}(i)}{1+\kappa} \leq \frac{w_{t+1}^L}{w_{t+1}^H}, \ 0 \leq i < i_{t+1}^H, \tag{18}$$

$$\frac{\tilde{h}_{t+1}(i)}{1+\kappa} \geq \frac{w_{t+1}^L}{w_{t+1}^H},\ i_{t+1}^H \leq i \leq 1. \tag{19}$$

Whereas $\frac{\mathcal{H}_{t+1}(i^H)}{i^H}$ is a decreasing function of i^H, $\tilde{h}_{t+1}(i)$ is a non-decreasing function. Moreover,

$$\frac{1-\alpha}{\alpha}\frac{\mathcal{H}_{t+1}(i^H)}{i^H} > \frac{\tilde{h}_{t+1}(i^H)}{1+\kappa}$$

for sufficiently small $i^H > 0$ and

$$\frac{1-\alpha}{\alpha}\frac{\mathcal{H}_{t+1}(i^H)}{i^H} < \frac{\tilde{h}_{t+1}(i^H)}{1+\kappa}$$

for i^H sufficiently close to 1.

To find the time $t+1$ equilibrium pivotal dynasty i_{t+1}^H, it is sufficient to "solve" the following equation in i^H:

$$\frac{1-\alpha}{\alpha}\frac{\mathcal{H}_{t+1}(i^H)}{i^H} = \frac{\tilde{h}_{t+1}(i^H)}{1+\kappa} \tag{20}$$

If this equation has a solution, it is unique. Since $\tilde{h}_{t+1}(\cdot)$ may be discontinuous, the non-existence of a solution to equation (20) is possible. But even if (20) has no solution, there exists a unique i_{t+1}^H satisfying the following conditions:

$$\frac{1-\alpha}{\alpha}\frac{\mathcal{H}_{t+1}(i)}{i} > \frac{\tilde{h}_{t+1}(i)}{1+\kappa},\ 0 \leq i < i_{t+1}^H,$$

$$\frac{1-\alpha}{\alpha}\frac{\mathcal{H}_{t+1}(i)}{i} \leq \frac{\tilde{h}_{t+1}(i)}{1+\kappa},\ i_{t+1}^H \leq i \leq 1.$$

This i_{t+1}^H is the required time $t+1$ equilibrium pivotal dynasty

As for $h_{t+1}(\cdot)$, it is determined as follows:

$$h_{t+1}(i) = 0,\ 0 \leq i < i_{t+1}^H,$$

$$h_{t+1}(i) = \tilde{h}_{t+1}(i),\ i_{t+1}^H \leq i \leq 1. \ \square$$

It should be noticed that, unlike temporary equilibrium, non-uniqueness of equilibria does no appear in intertemporal equilibrium. This is because in the definition of temporary equilibrium at time t agents take the wage rates at time $t+1$ as given, whereas in intertemporal equilibrium they are determined endogenously. Also it is noteworthy that in intertemporal equilibrium an agent spend a positive fraction of his income on education if and only if his offspring will be human capital supplier on the labor market.

4. Steady-state equilibria

We now turn to the examination of steady-state equilibria. They are characterized by the feature that the wage rates and the fractions of educated agents supplying human capital on the labor market and uneducated agents supplying unskilled labor are constant over time and that inside each dynasty children find themselves in the same position as their parents.

At a steady-state equilibrium, the amount of human capital h^* supplied by agents from an educated dynasty depends only on the wage paid to one unit of human capital, w^{H*}, because h^* is the solution to the following equation: $h = \gamma(w^{H*})^\kappa \psi(h)$. Hence, at a steady-state equilibrium all educated dynasties supply the same amount of it. Therefore we can define steady-state equilibria as follows.

A couple (i^{H*}, h^*), $i^{H*} \in (0,1)$, $h^* > 0$, is called *a steady-state equilibrium* if the sequence $\{i_t, h_{t+1}(\cdot)\}_{t=0}^{\infty}$ given by

$$i_t = i^{H*},\ t = 0, 1, 2, \ldots,$$

$$h_{t+1}(i) = 0,\ 0 \le i < i^{H*},\ t = 0, 1, 2, \ldots,$$

$$h_{t+1}(i) = h^*,\ i^{H*} \le i \le 1,\ t = 0, 1, 2, \ldots,$$

is an equilibrium path starting from $h_0(\cdot)$ defined as follows:

$$h_0(i) = 0,\ 0 \le i < i^{H*},$$

$$h_0(i) = h^*,\ i^{H*} \le i \le 1.$$

It follows from (10) and (11) that at any steady state equilibrium (i^{H*}, h^*) the total supply of human capital, H^* and the wage rates of unskilled labor and human capital, w^{L*} and w^{H*}, are given as follows:

$$H^* = (1 - i^{H*})h^*,$$

$$w^{H*} = \alpha \left(\frac{i^{H*}}{H^*}\right)^{1-\alpha} = \alpha \left(\frac{i^{H*}}{(1 - i^{H*})h^*}\right)^{1-\alpha}, \tag{21}$$

$$w^{L*} = (1-\alpha)\left(\frac{H^*}{i^{H*}}\right)^{\alpha} = (1-\alpha)\left(\frac{(1-i^{H*})h^*}{i^{H*}}\right)^{\alpha}. \tag{22}$$

We are more precisely interested in the ratio of unskilled labor wage to human capital wage:

$$\frac{w^{L*}}{w^{H*}} = \frac{1-\alpha}{\alpha}\frac{(1-i^{H*})h^*}{i^{H*}}, \tag{23}$$

which is a decreasing function of i^{H*} and an increasing function of h^*. Also we are interested in the *skill premium* P^*, which is a reasonable measure of income inequality at a steady-state (i^{H*}, h^*) equilibrium of our model. It is defined as the proportion of the wage earned by an educated individual to the wage of an unskilled individual:

$$P^* := \frac{w^{H*}h^*}{w^{L*}}.$$

It follows from (23) that on the set of steady-state equilibria the skill premium can be considered as an increasing function of i^{H*}:

$$P^* = \frac{\alpha}{1-\alpha}\frac{i^{H*}}{1-i^{H*}}.$$

Let (i^{H*}, h^*) be a steady-state equilibrium and $\{i_t, h_{t+1}(\cdot)\}_{t=0}^{\infty}$ be the corresponding equilibrium path. It follows from (15)-(16) that for this path,

$$\tilde{h}_{t+1}(i) = \begin{cases} \gamma(w^{L*})^{\kappa}, & 0 \le i < i^{H*}, \\ \gamma(w^{H*})^{\kappa}\psi(h^*), & i^{H*} \le i \le 1. \end{cases}$$

Therefore (18) and (19) can be rewritten as respectively

$$\frac{\gamma(w^{L*})^{\kappa}}{1+\kappa} \le \frac{w^{L*}}{w^{H*}} \tag{24}$$

and

$$\frac{\gamma(w^{H*})^{\kappa}\psi(h^*)}{1+\kappa} \ge \frac{w^{L*}}{w^{H*}}. \tag{25}$$

The first of these inequalities means that the uneducated agents have no incentives to spend money on the education of their offsprings and the second that the educated agents have such incentives. It is also clear that

$$h^* = \gamma(w^{H*})^{\kappa}\psi(h^*). \tag{26}$$

One can easily prove the following proposition.

Proposition 2. *A couple* (i^{H*}, h^*)*,* $0 < i^{H*} < 1$*,* $h^* > 0$*, is a steady-state equilibrium if and only if for* w^{H*} *and* w^{L*} *given by (21) and (22) respectively, (24)-(26) hold true.*

Let us now describe the relationship between the share of uneducated agents in the population, i^{H*}, and the human capital accumulated by an educated agent in a steady-state equilibrium, h^*. It is reasonable to conjecture that this relationship is increasing because a higher fraction of uneducated agents can lead to a larger skill premium and wages of educated individuals and hence to higher individual educational expenditures. The following lemma says that this conjecture is true.

Lemma 2. *There is a smooth increasing function* $\chi : (0,1) \to \mathbb{R}_+$ *and numbers* L_1 *and* L_2*,* $0 < L_2 < L_1 \leq 1$*, such that for any* $i^* \in (0,1)$ *and for* w^{H*} *and* w^{L*} *given by (21) and (22) respectively,*

(26) is equivalent to

$$h^* = \chi(i^{H*}), \tag{27}$$

(25) is equivalent to $i^{H*} \geq L_2$*,*

(24) is equivalent to $i^{H*} \leq L_1$*.*

The proof of this lemma is relegated to Appendix.

The following theorem describing the the structure of steady-state equilibria follows directly from Proposition 2 and Lemma 2.

Theorem 3. *There is a smooth increasing function* $\chi : (0,1) \to \mathbb{R}_+$ *and numbers* L_1 *and* L_2*,* $0 < L_2 < L_1 \leq 1$*, such that a couple* (i^{H*}, h^*) *is a steady-state equilibrium if and only if either*

$$L_2 \leq i^{H*} \leq L_1 \ (\textit{if } L_1 < 1)$$

or

$$L_2 \leq i^{H*} < L_1 \ (\textit{if } L_1 = 1)$$

and

$$h^* = \chi(i^{H*}).$$

Theorem 3 reads that, like in Ray (2006), the set of steady-state equilibria is a continuum. More precisely, this set is essentially an interval which can be parameterized by the fraction of uneducated agents in the population.

Another interesting parametrization of the set of steady-state equilibria is that by the skill premium. This parametrization can help us to explain why all equilibrium values of the fraction of uneducated agents lies in the interval $[L_2, L_1]$. If $i^{H*} < L_2$, then the couple $(i^{H*}, \chi(i^{H*})$ is not a steady-state equilibrium because the wage rate of unskilled labor is so high and the skill premium is so small that even the educated parents have no incentives to spend a positive fraction of their incomes on the education of their children. If $L_2 \leq i^{H*} \leq L_1$, then the skill premium is such that the educated individuals prefer to see their children educated while the uneducated agents find it too expensive to spend money on the education of their children. Finally, if $i^{H*} > L_1$, the couple $(i^{H*}, \chi(i^{H*})$ is not a steady-state equilibrium because the wage rate of unskilled labor is so small and the skill premium is so high that even the uneducated individuals are ready to spend money on the education of their children.

Let us now consider the question of what is the relationship between inequality, measured by the skill premium, and output on the set of steady-state equilibria. To sketch the broad outlines of this relationship, it is sufficient to look at the dependence of output on i^{H*}, because the skill premium P^* is an increasing function of i^{H*}. The level of output, Y^*, corresponding to a steady-state equilibrium $(i^{H*}, \chi(i^{H*}))$ is

$$Y^* = [(1 - i^{H*})\chi(i^{H*})]^{\alpha}[i^{H*}]^{1-\alpha}.$$

It would be difficult to derive an analytical form of the function $[(1 - i)\chi(i)]^{\alpha}[i]^{1-\alpha}$. The shape of the graph of this function depends on the parameters of the model, α and κ. However, it is clear that the dependence of Y^* on i^{H*} is quite ambiguous. Our computational experiments show that this dependence on the interval $[L_2, L_1]$ can be of an inverted U-shaped form or increasing.

On Fig. 1 we present L_1, L_2 and the graph of $[(1 - i)\chi(i)]^{\alpha}[i]^{1-\alpha}$ on the segment $[0, 1]$ at $\alpha = 0.4$, $\kappa = 0.5$. On $[L_2, L_1]$ the function $[(1 - i)\chi(i)]^{\alpha}[i]^{1-\alpha}$ has an inverted U-shaped form.

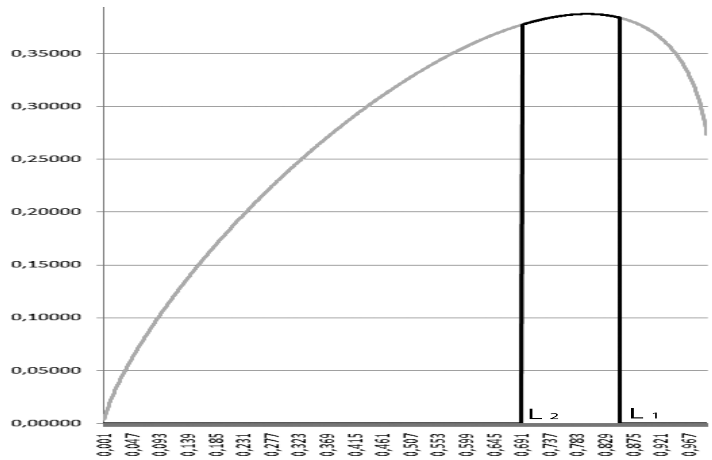

Figure 1.

On Fig. 2 we present L_1, L_2 and the graph of $\Gamma(i)$ on the segment $[0, 1]$ at $\alpha = 0.3$, $\kappa = 0.96$. On $[L_2, L_1]$ the function $[(1-i)\chi(i)]^{\alpha}[i]^{1-\alpha}$ increasing.

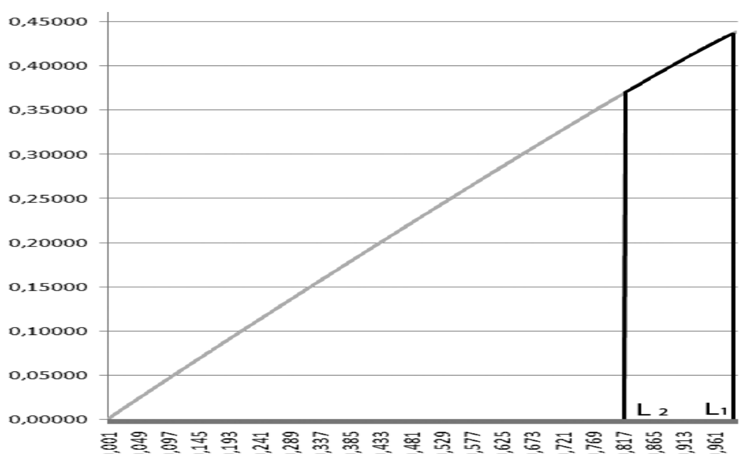

Figure 2.

5. Convergence of equilibrium paths

The following theorem reads that any equilibrium path converge to a steady-state equilibrium and that the number of uneducated agents does not increase in time (except, perhaps, at time $t = 1$).

Theorem 4. *For any equilibrium path* $\{i_t^H, h_{t+1}(\cdot)\}_{t=0}^{\infty}$ *the sequence* $\{i_t^H\}_{t=1}^{\infty}$ *is non-increasing (it may be that* $i_1^H > i_0^H$*) and there is a steady-state equilibrium* (i^{H*}, h^*) *such that*

$$i_t^H \longrightarrow_{t\to\infty} i^{H*},$$

$$h_t(i) \longrightarrow_{t\to\infty} h^*, \; i_t^H \leq i \leq 1.$$

Proof. It is sufficient to prove that for any equilibrium path $\{i_t^H, h_{t+1}(\cdot)\}_{t=0}^{\infty}$ the sequence $\{i_t^H\}_{t=1}^{\infty}$ is non-increasing (it may be that $i_1^H > i_0^H$).

Let $\{i_t^H, h_{t+1}(\cdot)\}_{t=0}^{\infty}$ be an equilibrium path. Show that for any $t = 1, 2, \ldots$,

$$i_{t+1}^H \leq i_t^H.$$

We have

$$\tilde{h}_{t+1}(i) = \gamma \left(w_t^H\right)^{\kappa} \psi(h_t(i)), \; i_t^H \leq i \leq 1.$$

Therefore,

$$\mathcal{H}_{t+1}(i_t^H) = \gamma \left(w_t^H\right)^{\kappa} \int_{i_t^H}^{1} \psi(h_t(i))di,$$

where $\mathcal{H}_{t+1}(\cdot)$ is defined by (17).

It is clear that $\frac{\psi(h)}{h}$ decreases as $h > 0$ increases and hence

$$\frac{\psi[h_t(i)]}{h_t(i)} \leq \frac{\psi[h_t(i_t^H)]}{h_t(i_t^H)}, \; i_t^H \leq i \leq 1.$$

It follows that

$$\psi[h_t(i)] \leq \frac{\psi[h_t(i_t^H)]}{h_t(i_t^H)} h_t(i), \; i_t^H \leq i \leq 1.$$

Therefore,

$$\mathcal{H}_{t+1}(i_t^H) \leq \gamma \left(w_t^H\right)^{\kappa} \frac{\psi[h_t(i_t^H)]}{h_t(i_t^H)} \int_{i_t^H}^{1} h_t(i)di = \tilde{h}_{t+1}(i_t^H) \frac{\mathcal{H}_t(i_t^H)}{h_t(i_t^H)}.$$

Thus,

$$\frac{\mathcal{H}_{t+1}(i_t^H)}{\tilde{h}_{t+1}(i_t^H)} \leq \frac{\mathcal{H}_t(i_t^H)}{h_t(i_t^H)},$$

which implies $i_{t+1}^H \leq i_t^H$.

To complete the proof, it is sufficient to note that the sequence $\{i_t^H\}_{t=1}^\infty$ is bounded from below and therefore converges to some i^{H*}. It is no difficult to check that $i^{H*} > 0$ and the required steady-state equilibrium is the couple (i^{H*}, h^*), where $h^* = \chi(i^{H*})$. $\square$

6. Endogenous growth

In this section we propose an endogenous growth version of our model. To do this, we introduce an endogenously formed variable reflecting the state of technology at time t, A_t. An increase in its value leads (i) to a higher effectiveness of unskilled labor and (ii) promote accumulation of human capital. In its turn, the accumulation of human capital contribute to the growth of the value of this variable through a macroeconomic externality.

More precisely, our assumptions are as follows. The output at time t, Y_t, is given by

$$Y_t = H_t^\alpha (A_t L_t)^{1-\alpha}.$$

Therefore the wage earned by each agent supplying unskilled labor on the labor market is $w_t^L = (1-\alpha) H_t^\alpha (A_t L_t)^{1-\alpha}$.

The human capital of an agent of dynasty i at time t, $h_t(i)$, depends not only on the human capital of his parent, $h_{t-1}(i)$ and the amount of money the parent spent for his higher education, $e_{t-1}(i)$, but also on the state of technology at time $t-1$, A_{t-1}:

$$h_t(i) = e_{t-1}(i)^\kappa (h_{t-1}(i) + A_{t-1})^{1-\kappa}, \quad 0 < \kappa < 1. \tag{28}$$

As for the formation of A_t, we assume that

$$A_t = \Phi(H_{t-1}, A_{t-1}),$$

where $\Phi : \mathbb{R}_+^2 \to \mathbb{R}$ is a continuous homogeneous of degree one concave function.

Thus, the variable A_t $i)$ shows the efficiency of unskilled labor and $ii)$ plays the role of an input in the educational production function. Its value can grow over time through the accumulation of human capital.

The behavior of individuals is the same as in the case of the exogenous growth model with the only difference that (1) is replaced by (28). The temporal and intertemporal equilibria are also defined in practically the same way as above. The difference is that a temporal equilibrium at each time t is described not by a couple $\{i_t^H, h_{t+1}(\cdot)\}$, but by a triple $\{i_t^H, A_{t+1}, h_{t+1}(\cdot)\}$ satisfying the requirements formulated in Subsection 3.1 and the equation $A_{t+1} = \Phi(\int_0^1 h_t(i)di, A_t)$. Clearly, an initial state of an intertemporal equilibrium path is determined by a couple $\{A_0, h_0(\cdot)\}$. The existence of an intertemporal equilibrium paths is also proved in the same way.

As for *a steady-state equilibrium*, it is defined as a triple $\{i^{H*}, 1+g^*, h^*\}$, $i^{H*} \in (0,1)$, $h^* > 0$, such that the sequence $\{i_t, A_{t+1}, h_{t+1}(\cdot)\}_{t=0}^{\infty}$ given by

$$i_t = i^{H*},\ t = 0,1,2,\ldots,$$

$$A_t = (1+g^*)^t,\ t = 0,1,2,\ldots,$$

$$h_{t+1}(i) = 0,\ 0 \le i < i^{H*},\ t = 0,1,2,\ldots,$$

$$h_{t+1}(i) = A_{t+1}h^*,\ i^{H*} \le i \le 1,\ t = 0,1,2,\ldots,$$

is an equilibrium path starting from the initial state $\{A_0, h_0(\cdot)\}$ given by $A_0 = 1$,

$$h_0(i) = 0,\ 0 \le i < i^{H*},$$

$$h_0(i) = h^*,\ i^{H*} \le i \le 1.$$

As in the case of exogenous growth, it is not difficult to show that *any equilibrium path converges to a steady-state equilibrium.* The structure of the set of steady-state equilibria is described in the following theorem.

Theorem 5. *There is a smooth increasing function $\chi : (0,1) \to \mathbb{R}_+$ and numbers L_1 and L_2, $0 < L_2 < L_1 \le 1$, such that a triple $(i^{H*}, 1+g^*, h^*)$ is a steady-state equilibrium if and only if either*

$$L_2 \le i^{H*} \le L_1\ (\textit{if } L_1 < 1)$$

or

$$L_2 \le i^{H*} < L_1\ (\textit{if } L_1 = 1)$$

and

$$h^* = \chi(i^{H*}),\ 1+g^* = \Phi((1-i^{H*})h^*, 1)$$

What does the endogenous growth version of our model tell about the nexus of income inequality and the rate of economic growth? The debate on this issue is not settled. The classical approach suggests that inequality stimulates capital accumulation and thus promotes economic growth, whereas the modern approach argues in contrast that for sufficiently wealthy economies equality stimulates investment in human capital and hence may enhance economic growth.

In our model, the nature of the relationship between income inequality and the rate of growth on the set of steady-state equilibria can be illustrated by the graph of the dependence of $(1 - i^{H*})\chi(i^{H*})$ on i^{H*} because, on the one hand, the rate of growth is an increasing function of $(1-i^{H*})\chi(i^{H*})$ and, on the other hand, the skill gap P^* increases with an increase in i^{H*}. Our simulations show that this dependence of $(1 - i^{H*})\chi(i^{H*})$ on i^{H*} on the interval $[L_2, L_1]$ can increasing ($\alpha = 0.4$,$\kappa = 0.99$) or decreasing ($\alpha = 0.2$,$\kappa = 0.3$) or of an inverted U-shaped form ($\alpha = 0.8$,$\kappa = 0.9$).

In our model, the nature of the relationship between income inequality and the rate of growth on the set of steady-state equilibria can be illustrated by the graph of the dependence of $(1 - i^{H*})\chi(i^{H*})$ on i^{H*} because, on the one hand, the rate of growth is an increasing function of $(1-i^{H*})\chi(i^{H*})$ and, on the other hand, the skill gap P^* increases with an increase in i^{H*}. Our computational experiments show that this dependence of $(1-i^{H*})\chi(i^{H*})$ on i^{H*} on the interval $[L_2, L_1]$ can be increasing, decreasing or of an inverted U-shaped form.

On Figs. 3-5 we present graphs of $(1 - i)\chi(i)$ on the segment $[0, 1]$ and indicate L_2 and L_1 for different values of parameters α and κ.

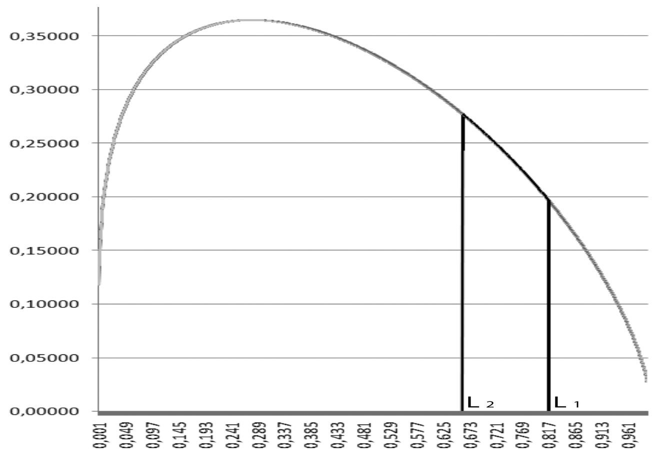

Figure 3. $\alpha = 0.4$, $\kappa = 0.5$.

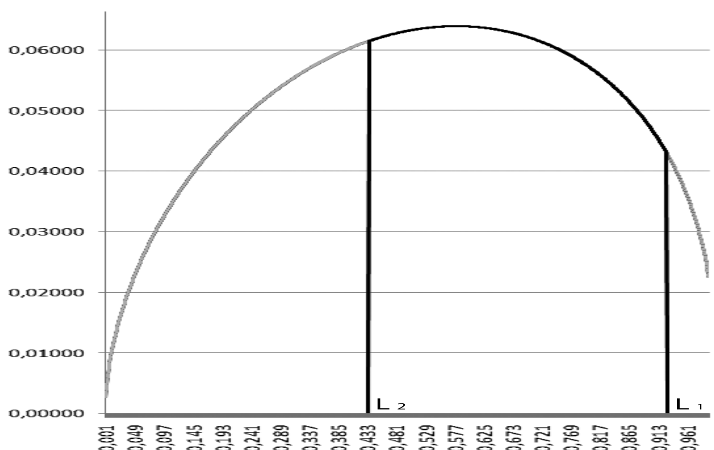

Figure 4. $\alpha = 0.7$, $\kappa = 0.8$.

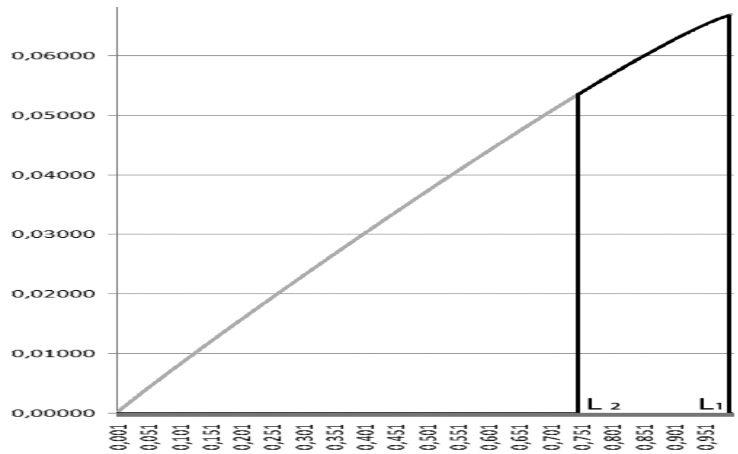

Figure 5. $\alpha = 0.3$, $\kappa = 0.96$.

7. Conclusion

To answer the question of how the growth process affects inequality, namely the question of the feedback of growth into the evolution of inequality across time, we developed a model characterized both by (i) capital accumulation, out of a family altruism motive, and (ii) a twofold occupational choice between skilled and unskilled position.

We showed how the initial wage distribution evolves across time. The pivotal dynasty splitting the population in two classes evolves in the direction of including more and more families in the skilled-labor class. However in the long run, wage inequality and the division of population between skilled and unskilled remains.

The steady state equilibrium itself is actually a continuum and for each of these steady state a different wage inequality and splitting prevails. We show that the relationship between the location of the steady state pivotal dynasty, and hence the long run wage inequality, and the level of output (or the rate of growth of output) is ambiguous.

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Appendix A. Proof of Lemma 2

To describe the set of steady-state equilibria we verify two things. First we show that on the set of steady-state equilibria there is a monotonically decreasing relationship between the equilibrium pivotal dynasty i^{H*}, which shows the fraction of uneducated agents in the population, and the amount of human capital, h^*, supplied by each educated agent. Second, we show that the set of equilibrium values of i^H is an interval.

Let us first prove the following lemma.

Lemma 6. 1) *There is a smooth increasing function* $\chi : (0,1) \to \mathbb{R}_+$ *such that for any* $i \in (0,1)$,

$$h = \gamma \left[\alpha \left(\frac{i}{(1-i)h} \right)^{1-\alpha} \right]^{\kappa} \psi(h) \Leftrightarrow h = \chi(i). \tag{A.1}$$

This function satisfies

$$\chi(i) \to 0 \text{ as } i \to 0 \qquad \text{and} \qquad \chi(i) \to \infty \text{ as } i \to 1. \tag{A.2}$$

2) $\frac{1-i}{i}\chi(i)$ *monotonically decreases from* ∞ *to* $(\gamma\alpha^{\kappa})^{\frac{1}{(1-\alpha)\kappa}}$ *as* i *increases from* 0 *to* 1.

Proof. 1) Given that $\psi(h) = h^{\kappa}(h+1)^{1-\kappa}$, we can rewrite the first equation in $(A.1)$ as

$$h = \gamma\alpha^{\kappa} \left(\frac{i}{1-i} \right)^{(1-\alpha)\kappa} h^{\alpha\kappa}(h+1)^{1-\kappa},$$

or, after dividing both sides by $h^{\alpha\kappa}(h+1)^{1-\kappa}$, as

$$\frac{h^{1-\alpha\kappa}}{(h+1)^{1-\kappa}} = \gamma\alpha^{\kappa} \left(\frac{i}{1-i} \right)^{(1-\alpha)\kappa}. \tag{A.3}$$

The LHS of the last equation is continuous and increasing in h, tends to 0 as $h \to 0$ and to $+\infty$ as h tends to $+\infty$ since $1 - \alpha\kappa > 1 - \kappa$. The RHS of this equation is continuous and increasing in i, tends to 0 as $i \to 0$ and to $+\infty$ as $i \to 1$. It is then obvious that for any i there exists a solution to (A.3) in h. To complete the proof, denote this solution by $\chi(i)$ and notice that the both properties in (A.2) hold true.

2) After some rearrangement of (A.3) we can get

$$\frac{1-i}{i}\chi(i) = (\gamma\alpha^{\kappa})^{\frac{1}{(1-\alpha)\kappa}} \left(1 + \frac{1}{\chi(i)} \right)^{(1-\kappa)/(1-\alpha)\kappa}.$$

Since $\chi(i)$ monotonically increases from 0 to ∞ , $\frac{1-i}{i}\chi(i)$ monotonically decreases from ∞ to $(\gamma\alpha^{\kappa})^{\frac{1}{(1-\alpha)\kappa}}$ as i increases from 0 to 1. $\square$

Because of (21) and Lemma 6, equation (26) can be rewritten as (27), where $\chi(\cdot)$ is the function introduced in Lemma 6. Thus, the amount of human capital supplied by each educated agent is an increasing function of the fraction of uneducated agents in the population.

Also, by (23) and (26), we can rewrite (25) as

$$\frac{h^*}{1+\kappa} \geq \frac{1-\alpha}{\alpha}\frac{(1-i^{H*})h^*}{i^{H*}},$$

or, equivalently, as

$$i^{H*} \geq L_2 := \frac{(1-\alpha)(1+\kappa)}{(1-\alpha)(1+\kappa)+\alpha}.$$

Let us now rewrite (24) as

$$\frac{\gamma}{1+\kappa}(1-\alpha)^\kappa \left(\frac{(1-i^{H*})h^*}{i^{H*}}\right)^{\alpha\kappa} \leq \frac{1-\alpha}{\alpha}\frac{(1-i^{H*})h^*}{i^{H*}},$$

This inequality can be re-written as

$$\frac{\gamma}{1+\kappa}(1-\alpha)^\kappa \leq \frac{1-\alpha}{\alpha}\left(\frac{1-i^{H*}}{i^{H*}}h^*\right)^{1-\alpha\kappa} = \frac{1-\alpha}{\alpha}\left(\frac{1-i^{H*}}{i^{H*}}\chi(i^{H*})\right)^{1-\alpha\kappa}$$

or, after substituting (27) as

$$\frac{\gamma}{1+\kappa}(1-\alpha)^\kappa \leq \frac{1-\alpha}{\alpha}\left(\frac{1-i^{H*}}{i^{H*}}\chi(i^{H*})\right)^{1-\alpha\kappa}. \tag{A.4}$$

By Lemma 6, $\frac{1-i}{i}\chi(i)$ monotonically decreases from ∞ to $(\gamma\alpha^\kappa)^{\frac{1}{(1-\alpha)\kappa}}$ as i increases from 0 to 1. Therefore, if

$$\left(\frac{\alpha\gamma}{1+\kappa}(1-\alpha)^{\kappa-1}\right)^{\frac{1}{1-\alpha\kappa}} \geq (\gamma\alpha^\kappa)^{\frac{1}{(1-\alpha)\kappa}},$$

then (A.4) is equivalent to

$$i^{H*} \leq L_1, \tag{A.5}$$

where L_1 is the solution to the following equation in i:

$$\frac{\gamma}{1+\kappa}(1-\alpha)^\kappa = \frac{1-\alpha}{\alpha}\left(\frac{1-i}{i}\chi(i)\right)^{1-\alpha\kappa}.$$

If

$$\left(\frac{\alpha\gamma}{1+\kappa}(1-\alpha)^{\kappa-1}\right)^{\frac{1}{1-\alpha\kappa}} < (\gamma\alpha^\kappa)^{\frac{1}{(1-\alpha)\kappa}},$$

then (A.4) holds for all $i^{H*} \in (0,1)$.

To complete the the proof of Lemma 2, it is necessary to show that $L_2 < L_1$. To do this, let

$$h^{**} := \chi(L_2),$$

$$w^{H**} := \alpha \left(\frac{L_2}{(1-L_2)h^{**}} \right)^{1-\alpha}, \; w^{L**} := (1-\alpha) \left(\frac{(1-L_2)h^{**}}{L_2} \right)^{\alpha}.$$

We have

$$\frac{h^{**}}{1+\kappa} = \frac{\gamma(w^{H**})^{\kappa}\psi(h^{**})}{1+\kappa} = \frac{w^{L**}}{w^{H**}}.$$

It follows that

$$w^{H**}h^{**} > \frac{w^{H**}h^{**}}{1+\kappa} = w^{L**}.$$

Hence

$$\frac{\gamma(w^{L**})^{\kappa}}{1+\kappa} < \frac{\gamma(w^{H**}h^{**})^{\kappa}}{1+\kappa} < \frac{\gamma(w^{H**})^{\kappa}(h^{**})^{\kappa}(h^{**}+1)^{1-\kappa}}{1+\kappa}$$
$$= \frac{\gamma(w^{H**})^{\kappa}\psi(h^{**})}{1+\kappa} = \frac{w^{L**}}{w^{H**}}.$$

It is clear that (24) fulfills as a strict inequality if and only if (A.5) fulfills as a strict inequality. Therefore the last chain on inequalities implies $L_2 < L_1$. $\square$