Predatory Lending

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“Predatory Lending”

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Abstract

This paper studies the equilibrium predatory practices that may arise when the borrowers have behavioral weaknesses. Rational lenders offer short term contracts that can be renewed at the cost of paying a penalty fee. We show how the optimal contracts depend on the degree of naïveté of the time inconsistent customers. Penalty fees have a dual role : they increase market share by providing a useful commitment device to time-inconsistent but otherwise rational borrowers ; they are also a source of revenue from the semi-naïve borrowers who understand the need for commitment but fail to forecast their future time discount factor. We also show that perfect competition does not eliminate predatory practices, since the equilibrium contract entails a subsidized (below marginal cost) short-term loan that can only be profitable if a fraction of the borrowers end up paying the penalty fee.

JEL : D03, D18, D49, D86

Keywords : hyperbolic discounting, time inconsistency, sophistication, partial naïveté, exploitative contracts, credit cards

1 Introduction

The paper focuses on the equilibrium predatory practices that may arise when the borrowers have behavioral weaknesses. Our main assumption is that the borrowers have the $(\delta, \beta, \tilde{\beta})$ quasi-hyperbolic preferences introduced by O'Donoghue and Rabin (1999a, 2001). This assumption has two components. First, the quasi-hyperbolic preferences imply that the borrowers have a self-control problem (their inability to resist the urges of immediate consumption), which may lead to time-inconsistent choices, since their preferences are time-varying. Second, the borrowers are not fully aware of the dynamics of their preferences (they tend to overestimate their future discount factor). This second hypothesis, which we label the naïveté hypothesis will turn out to be a crucial element of the analysis.

The lending institutions are fully rational. They are aware of the behavioral weaknesses of their customers and will devise contracts that take advantage of those weaknesses. The

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purpose of the paper is to make a systematic exploration of those contracts, which we label exploitative contracts. We assume that the lending institutions offer short-terms lending contracts that can be renewed, at a cost. The contracts have therefore two main characteristics : the interest rate R and the roll-over fee F.

We show that the optimal exploitative contracts depend crucially on the degree of naïveté of the borrowers. Sophisticated individuals (those who are fully aware of the dynamics of their preferences) cannot be fooled. They are offered contract with positive fees, where the fees are used as a commitment device (that allows them to implement their optimal consumption path). This is no longer the case with naïve borrowers. Fully naïves don't understand the commitment value of the fees, and therefore focus only on the interest rate (since they don't expect to pay the fee). The optimal contract is one with high interest rates and zero fees. Partially naïves understand the commitment value of the fee, but can nevertheless be fooled, since they overestimate their future discount factor. The optimal contract is one with high interest rates and a positive fee, that the borrowers end-up paying in some configurations. Deceitful contracts thus only arise with partially naïves borrowers in the monopoly case.

Most authors argue that competition should eliminate the predatory practices. We show that this is not the case in our framework. Perfect competition allows the sophisticated consumers to take the whole surplus (the equilibrium interest rate falls to marginal cost and the firm's profits are zero at equilibrium). But predatory contracts do persists as long as a there is a small fraction of naïves. The optimal equilibrium contract is clearly based on deceit, since the firms offer short term loans at discount rates (rates below marginal cost) that can only be profitable if a fraction of customers roll over the loan and thus end up paying the penalty fee they did not expect to pay. Perfect competition implies that the lending firms do not profit from their predatory practices in equilibrium (profits are null in equilibrium). The main beneficiaries of the deceitful contracts are the sophisticated borrowers who benefit from the discount rates, and use the penalty fee as a commitment device.

This paper is related to the growing literature on contracting with boundedly rational agents which started with the seminal contribution of Della Vigna and Malmendier (2004) (henceforth referred as D&M). Their model is a three period version of the (δ, β, $\tilde{\beta}$) model of O'Donoghue and Rabin (1999a, 2001). They show that the optimal two-part contract depends on the nature of the good provided by the firm. They distinguish two types of goods : *leisure good*, which provides current benefits, but have future costs and *investment good*, that have current costs and future benefits. A monopoly that sells a *leisure good* should choose a low entry cost and a high usage cost (i.e. above marginal cost) to take advantage of the naïve's tendency to underestimate their future usage. If the firm sells an *investment good*, the optimal pricing policy is the converse : a high fixed cost and a low usage cost (below marginal cost), in order to exploit the semi-naïves tendency tendency to overestimate their future usage as well as their demand for commitment (which is provided by the high entry cost).

Heidhues and Kőszegi (2009) focus on over-commitment. Semi-naïves individuals who want to avoid the future consumption of an harmful good (a leisure good using D&M terminology), have the opportunity to buy a costly commitment technology. But most of them end up buying the good anyway as they underestimate their lack of self-control. The authors show that higher sophistication (greater awareness of the self control problem) is often worse than full naïveté, since it increases the spending on commitment without preventing the consumption of the harmful good. Heidhues and Kőszegi (2010) studies exploitative contracts in the credit card and mortgage market. Even though they assume full competition, their results are similar to D&M : the equilibrium contracts have low teaser rates for short term loans and large penalty fees for those who fall behind on their payment schedule.

The subject of Gottlieb (2008) is competition over time-inconsistent consumers. They point that the impact of competition on the pricing of leisure goods depends crucially on the customer's switching costs. D&M pricing (lump-sum transfers to bait customers and above marginal cost usage price) requires exclusive contracts. When those contracts are not feasible, firms choose marginal cost pricing (and zero lump-sum transfers).

Others significant contributions are Eliaz and Spiegler (2006) and Gabaix and Laibson (2006). Both show how firms may profit by devising contracts that discriminate between sophisticated and naïve customers. In the Eliaz & Spiegler setting, the agents differ in their ability to forecast the change in their future tastes. The optimal menu of contract provides perfect commitment devices for the sophisticated consumers, and exploitative contracts for the naïve. Gabaix & Laibson analyze the issue of informational shrouding. Firms offer a base good and and add-ons. They show how firms may exploit myopic consumers through marketing schemes that offers cheap base goods and shroud high-priced add-ons, and how sophisticated consumers profit from those schemes. They also show that competition does not eliminate shrouding.

Empirical evidence on the pricing of investment good is provided by Della Vigna and Malmendier (2006) in the context of the health club markets. Della Vigna and Malmendier (2004) also gives (second hand) evidence of the pricing of leisure goods with switching costs (credit cards, mobile phones). Gottlieb (2008) provides evidence of the pricing on leisure goods with switching costs in competitive industries (Tobacco and and alcohol industries). Gabaix and Laibson (2006) provide evidence on shrouding in the retail banking industry and in the printer market. First hand evidence on the practices of the credit card industry can be found in Ausubel (1991) and Shui and Asubel (1995) A full presentation of the payday loan market can be found in Stegman (2007) and Agarwal et al. (2009).

The paper proceeds as follows. Section 2 presents the model. Sections 3 and 4 analyze the optimal pricing strategy under monopolistic conditions. The form and the impact of the contract depend crucially on the degree of naïveté of the borrowers. Section 3 establishes the optimal contract for sophisticated borrowers, whereas section 4 show how firms can take advantage of the borrower's naïveté. Section 5 analyzes the effect of competition on the equilibrium pricing strategies. Section 6.1 studies the impact of predatory lending on the welfare of the borrowers under monopoly and competition. Section 7 concludes. All proofs omitted in the main text are given in the appendix.

2 The Model

2.1 Intertemporal Preferences

We assume that the borrowers have $(\delta, \beta, \tilde{\beta})$ quasi-hyperbolic intertemporal preferences At time t, the present value of the flow of future utilities $(u_s)_{s\geq t}$ is :

$$u_t + \beta \sum_{s\geq t+1} \delta^{s-t} u_s \tag{1}$$

where $\delta \leq 1$ is the long-run discount factor and $\beta \leq 1$ is the short-run one. $\beta < 1$ results from a self-control problem (the inability to resist to the urges of immediate consumption) Quasi-hyperbolic preferences (henceforth referred as q-hyper pref) imply that discounting is time-varying, since the discount factor between $t+1$ and $t+2$ is δ at time t and $\beta\delta$

at time $t+1$. Time consistency thus requires $\beta = 1$. β can therefore be considered as a measure of the degree of time inconsistency of the agent.

The parameter $\beta \leq \tilde{\beta} \leq 1$ is the expected short-run discount factor. A naïve agent overestimate his future discount factor (he expects - erroneously - to have the discount function $1, \tilde{\beta}\delta, \tilde{\beta}\delta^2, ..$ in all future periods). An agent with $\tilde{\beta} = 1$ is said to be fully naïve, whereas an agent with $\tilde{\beta} = \beta$ is a sophisticated one. $\tilde{\beta} - \beta$ is therefore a measure of the degree of the agent's naïveté (the tendency to believe that his lack of self-control is temporary)

For the sake of simplicity we assume from now on that : (i) individuals live three periods ; (ii) their instantaneous utility is linear ; and (iii) the long-run discount factor is $\delta = 1$.

2.2 The lending institution

The lending institution is a fully rational profit maximizing monopoly. The firm has zero fixed and marginal costs (the lending institution can borrow at zero cost from the central bank, her discount rate is thus zero). The firm proposes two types of borrowing contracts : a short-run contract and a long run one.

- The short-run contract (henceforth referred as SR contract). The firm lends for one period (at interest rate R), and gives the individual the possibility to rollover the debt for one period more. The option to rollover is costly. An individual that chooses to rollover must pay a penalty F on the outstanding debt. The total cost of borrowing is therefore R for the borrowers that repay in time and $R(R+F)$ for the borrowers that choose to rollover. Profit is therefore $R-1$ per unit lent in the former case, and $R(R+F)-1$ in the latter case (recall that the firm's discount rate is zero).
- The long-run contract (LR). The firm lends for two periods at interest rate R (per period), which yield a profit equal to $R^2 - 1$.

The firm is fully aware of the self-control and naïveté problems. Contracts are designed to exploit them.

2.3 The distribution of (β, $\tilde{\beta}$)

The borrowers are heterogenous. There is a continuum of agents of size one. We assume that the short-run discount factor β are uniformly distributed on the support [β_0, 1], where β_0 is the short-run discount factor of the most impatient borrower.

We also assume that the expected short-run discount factor $\tilde{\beta}$ is an increasing function of β, with :

$$\tilde{\beta}(\beta) = \begin{cases} \bar{\beta} & \text{if} \quad \beta < \bar{\beta} \\ \beta & \text{if} \quad \beta \geq \bar{\beta} \end{cases} \tag{2}$$

This formulation implies that all individuals with $\beta < \bar{\beta}$ are naïves, whereas all individual with $\beta \geq \bar{\beta}$ are fully sophisticated. The naïveté and the lack of self-control are correlated since $\tilde{\beta} - \beta$ is decreasing in β.

3 Contracting with sophisticated borrowers

We assume in this section that all the borrowers are fully sophisticated ($\bar{\beta} = \beta_0$). We first study the optimal contract when penalties are not allowed. Then we show that penalties increase both the profit of the firm and the welfare of the borrowers.

3.1 The optimal contract when penalties are not allowed

At time $t = 1$, the net benefit of borrowing one unit is $(1 - \beta R)$ if the agent repays at time $t = 2$, and $(1 - \beta R^2)$ if the agent repays at time $t = 3$. One-period borrowing is thus better for the agent (the converse is true for the firm, who earns $R - 1$ with the short term contract and $R^2 - 1$ with the long term one).

All agents with $\beta \leq 1/R$ wish to borrow for one period. Yet, sophisticated agents know that their intertemporal preferences will change at time $t = 2$. The net benefit of deferring the repayment, which is $R - R = 0$ at time $t = 1$, will be, at time $t = 2$, $(1 - \beta R)$. Therefore all agents who choose to borrow for one period at time $t = 1$ will rollover at time $t = 2$, and end up paying R^2 at time $t = 3$. Since two-period borrowing is profitable only for agents with $\beta \leq 1/R^2$, all agents with $\beta > 1/R^2$ will choose not to borrow. Figure 1 represents the choice of the individuals in the $[\beta_0, 1]$ space.

Figure 1: The Choices of sophisticated borrowers ($F = 0$)

This implies that the interest rate chosen by the firm must not exceed $1/\sqrt{\beta_0}$ (since no one borrows when $\beta_0 R^2 > 1$), in which case the firm earns $R^2 - 1$ per unit lent to each agent in the interval $[\beta_0, 1/R^2]$. Assuming that the agents can borrow at most one unit [1], the firm's profit function $\Pi(R, F)$ writes :

$$\Pi(R, 0) = \begin{cases} (R^2 - 1)G(1/R^2) & \text{if} \quad 1 \leq R \leq 1/\sqrt{\beta_0} \\ 0 & \text{if} \quad R > 1/\sqrt{\beta_0} \end{cases} \tag{3}$$

where $G(\beta) = (\beta - \beta_0)/(1 - \beta_0)$ is the cumulative distribution function of β. It is easy to show that the profit function is concave in the interval $[1, 1/\sqrt{\beta_0}]$. The maximum profit is reached when $R = R_0^* \equiv (1/\beta_0)^{1/4}$.

3.2 The optimal contract with penalties

Introducing penalties will increase both the firm's profit and the average welfare of the borrowers.

The reason is that penalties are used as a commitment device by the borrowers. Recall that all borrowers with $\beta \in]1/R^2, 1/R[$ wish to borrow (since $1 - \beta R > 0$) but choose not to since they know that they will rollover, and rollover is not profitable for them. A penalty changes the net benefit of deferring repayment at $t = 2$, which is now $1 - \beta(R + F)$, which implies that rollover is no longer profitable for all agents with $\beta > 1/(R + F)$. Knowing this, all agents with $\beta \in [1/(R + F), 1/R]$ choose to borrow for one period. The behavior of the other agents is not modified. Figure 2 represents the choice of the sophisticated borrowers in the $[\beta_0, 1]$ space when F is positive.

[1]This assumption is not essential. All we need is that all agents borrow the same amount.

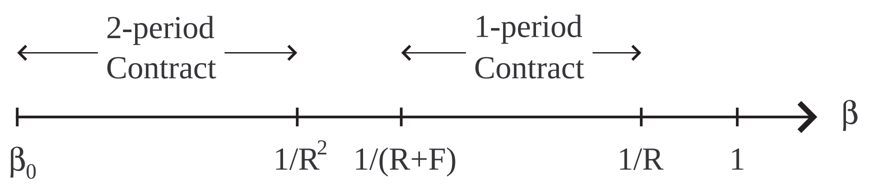

Figure 2: The Choices of sophisticated borrowers ($F > 0$)

Let's compute the profit, assuming that the interest rate is $R = R_0^*$ and that $1/(R_0^*)^2 \leq 1/(R_0^* + F)$. The profit of the firm is the sum of the profit made on the 2-period contracts $\Pi(R_0^*, 0)$ and the profit made on the one-period contract.

$$\Pi(R_0^*, F) = \Pi(R_0^*, 0) + (R-1)\left[G(1/R_0^*) - G(1/(R_0^* + F))\right] \tag{4}$$

which is clearly an increasing function of F. The maximum is reached when $1/(R_0^*)^2 = 1/(R_0^* + F) \Rightarrow F_0^* = R_0^*(R_0^* - 1)$. This results from the fact that choosing a penalty above F_0^* allows the individuals in the interval $[1/(R_0^* + F), 1/(R_0^*)^2]$ to replace their 2-period contract by one-period ones, which clearly reduces the profit of the firm by $(R_0^*)^2 - R_0^*$ per contract.

To compute the optimal (R_S, F_S) we must write the profit function. Assuming that $F < 1/\beta_0 - 1/\sqrt{\beta_0}$, the profit function writes :

$$\Pi(R,F) = \begin{cases} (R^2-1)G\left(\frac{1}{R+F}\right) + (R-1)\left[G\left(\frac{1}{R}\right) - G\left(\frac{1}{R+F}\right)\right] & \text{if} \quad 1 \leq R < R_F \\ (R^2-1)G\left(\frac{1}{R^2}\right) + (R-1)\left[G\left(\frac{1}{R}\right) - G\left(\frac{1}{R+F}\right)\right] & \text{if} \quad R_F \leq R < \frac{1}{\sqrt{\beta_0}} \\ (R-1)\left[G\left(\frac{1}{R}\right) - G\left(\frac{1}{R+F}\right)\right] & \text{if} \quad \frac{1}{\sqrt{\beta_0}} \leq R < \frac{1}{\beta_0} - F \\ (R-1)G\left(\frac{1}{R}\right) & \text{if} \quad \frac{1}{\beta_0} - F \leq R \leq \frac{1}{\beta_0} \end{cases} \tag{5}$$

where $R_F = (1 + \sqrt{1+4F})/2$ is the unique positive solution of the equation $1/R^2 = 1/(R+F)$. [2]

Proof. It is obvious that no one borrows for $R > 1/\beta_0$. One period contracts are chosen by all individuals when $1/\beta_0 - F \leq R \leq 1/\beta_0$. This results from the fact that they are profitable for all individuals (since $\beta R < 1$, $\forall \beta$) and that the borrowers know that they will not rollover (since $\beta(R+F) > 1$, $\forall \beta$). The profit is $(R-1)$ times the number of contracts $G(1/R)$. Note that 2-period contracts are nor an alternative since $\beta R^2 > 1$, $\forall \beta$. One-period contact are still the only alternative when $1/\sqrt{\beta_0} \leq R \leq 1/\beta_0 - F$, but they will be chosen only by a fraction of the individuals, since all the agents will β such that $\beta(R+F) \leq 1$ know that they will rollover. The profit is therefor $R-1$ times the number of contracts $G(1/R) - G(1/(R+F))$. Choosing $R \leq 1/\sqrt{\beta_0}$ allows the individual to choose between one-period and 2-period contracts. One-period contracts will be chosen by all those who don't expect to rollover (those with β such that $\beta(R+F) > 1$), the others will chose either a 2-period contract (if $\beta R^2 < 1$) or to abstain from borrowing (if $1/R^2 < \beta < 1/(R+F)$). The no-borrowing zone disappears when $1/R^2 > 1(R+F) \iff R > R_F$. Thus the profit is $R^2 - 1$ times the number of 2-period contracts ($G(1/R^2)$ when $R \geq R_F$, and $G(1/(R+F))$ when $R < R_F$) plus $R-1$ times the number of one-period contracts $G(1/R) - G(1/(R+F))$.

[2] The restriction $F < 1/\beta_0 - 1/\sqrt{\beta_0}$ ensures that R_F lies in the interval $[1, 1/\sqrt{\beta_0}]$. We show in appendix A.1 that the optimal F lies in the interval $[0, 1/\beta_0 - 1/\sqrt{\beta_0}]$.

As we can see in figure 3, the maximum profit is reached for $R_S = R_F$. The penalty is set at a level that : (i) ensures all agents choose either a one-period contract or a two-period contract ; (ii) avoids the cannibalization of 2-period contracts by one-period contracts.

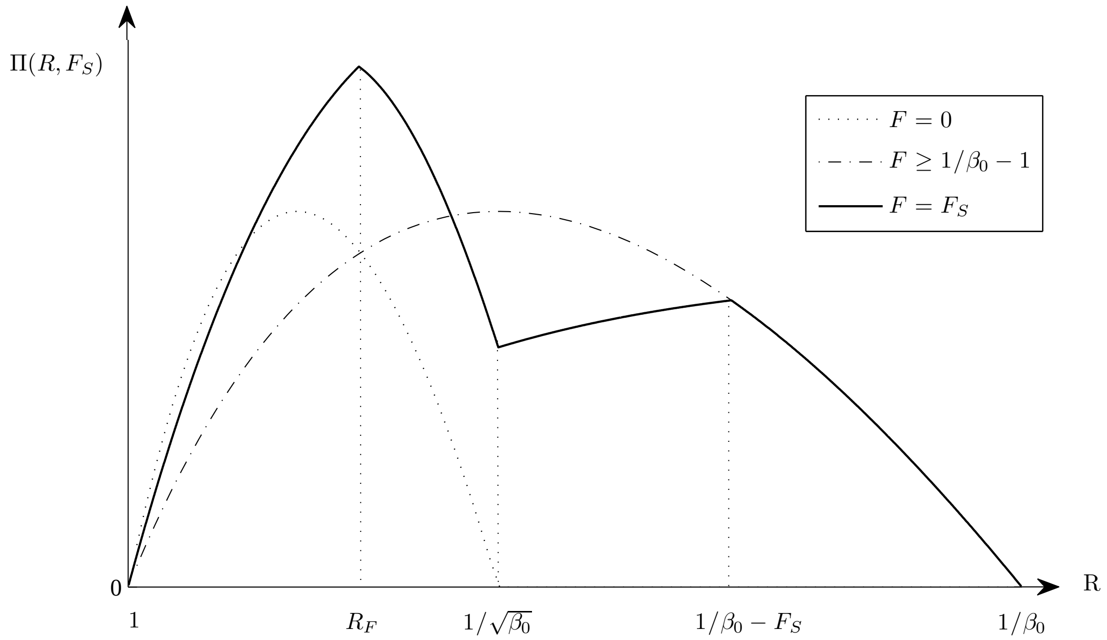

Figure 3: The profit function with sophisticated borrowers for $F = F_S$

In the appendix A.2, we show that the optimal (R_S, F_S) couple is :[3]

$$R_S = \left(\frac{1}{\beta_0}\right)^{1/3} \tag{6}$$

$$F_S = R_S(R_S - 1) = \left(\frac{1}{\beta_0}\right)^{1/3}\left(1 - \left(\frac{1}{\beta_0}\right)^{1/3}\right) \tag{7}$$

4 Contracting with naïve borrowers

We first we study the full naïveté case ($\bar{\beta} = 1$). Then we look at the partial naïveté case $\beta_0 < \bar{\beta} < 1$

4.1 Full naïveté

In this case the potential borrowers don't expect to rollover (since $\tilde{\beta}(R+F) = R+F > 1,\ \forall R > 1, F \geq 0$). Thus all the agents who wish to borrow one period (those with $\beta \leq 1/R$) will do so. Of course, all agents with $\beta < 1/(R+F)$ will rollover and pay the penalty. Now the firm must balance the additional income generated by the penalty fees and the loss of some 2-period borrowers due to the penalty (recall that the agents in the $[1/(R+F), 1/R]$ interval do not rollover). Figure 4 represents the choices of the individuals when $\beta_0 < 1/(R+F)$:

[3]The subscript $_S$ stands for sophisticated.

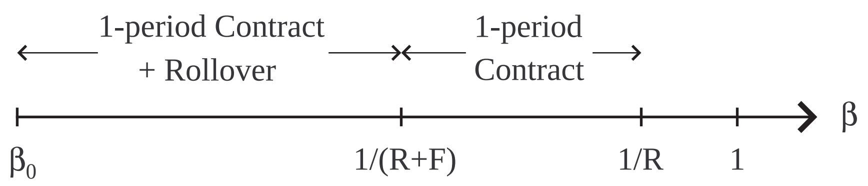

Figure 4: The Choices of fully naïve borrowers

The profit function of the firm writes :

$$\Pi(R,F)=\begin{cases}(R(R+F)-1)G\left(\frac{1}{R+F}\right)+(R-1)\left[G\left(\frac{1}{R}\right)-G\left(\frac{1}{R+F}\right)\right] & \text{if} \quad 1\leq R<\frac{1}{\beta_0}-F\\(R-1)G\left(\frac{1}{R}\right) & \text{if} \quad R\geq\frac{1}{\beta_0}-F\end{cases} \tag{8}$$

Proof. All agent with $\beta \leq 1/R$ borrow for one period. If $R \geq 1/\beta_0 - F$ the penalty is big enough to deter rollover for all agents ($\beta(R+F) > 1, \forall\beta$). The profit is (R-1) times the number of contracts $G(1/R)$. This is no longer the case for $R < 1/\beta_0 - F$. The agents whose $\beta < 1/(R+F)$ will rollover and pay the penalty. The profit is (R(R+F)-1) times the number individual who rollover $G(1/(R+F))$ plus $R-1$ times the number of individuals who don't $G(1/R) - G(1/(R+F))$.

It is easy to show that (i) the firm chooses a contract with zero penalty ($\tilde{F} = 0$); and (ii) the optimal interest rate R_N is such that $1/\sqrt{\beta_0} < R_N < 1/\beta_0$ (see the appendix for the proof). The figure 5 shows the optimal solution.

The logic of the no-penalty result is straightforward. Introducing a penalty F leads to a gain of F on the $G(1/(R+F))$ individuals that continue to rollover and a loss of $R^2 - R$ on the $G(1/R) - G(1/(R+F))$ individuals that no longer rollover. The net gain is :

$$\begin{aligned} F\times G\left(\frac{1}{R+F}\right)-R(R-1)\left[G\left(\frac{1}{R}\right)-G\left(\frac{1}{R+F}\right)\right] &= F\frac{\frac{1}{R+F}-\beta_0}{1-\beta_0}-R(R-1)\frac{\frac{1}{R}-\frac{1}{R+F}}{1-\beta_0}\\ &= -\frac{1}{1-\beta_0}\frac{F^2}{R+F}<0\end{aligned}$$

which is always negative. Adding a penalty is therefore never profitable when the borrowers are fully naïve.

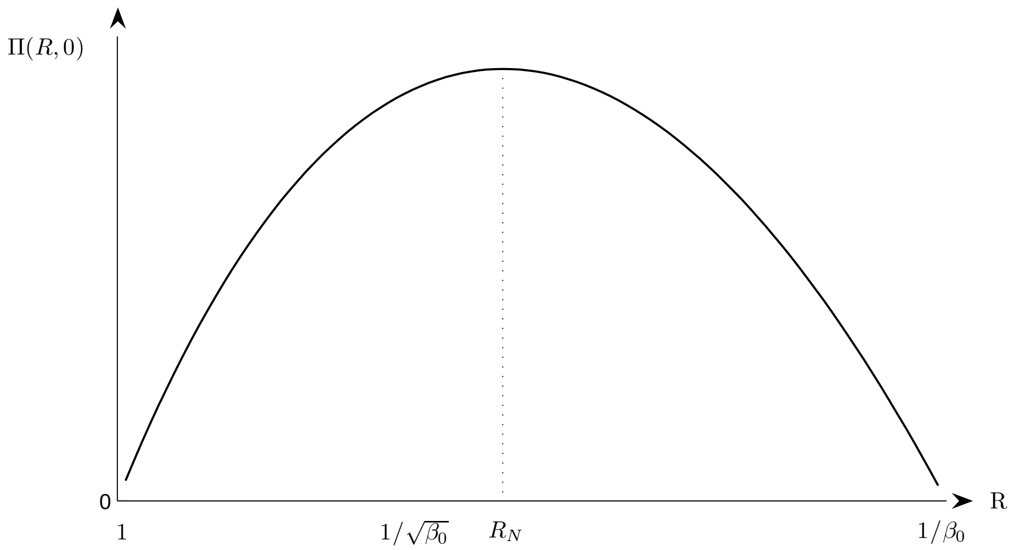

Figure 5: The profit function with fully naïve borrowers

4.2 Partial naïveté

There are three equilibrium configurations in the partial naïveté case, which we label : (i) nearly sophisticated ; (ii) sophisticated naïve ; and (iii) nearly naïve. In the following we characterize each configuration (full proofs are given in appendix C).

4.2.1 Configuration 1 : Nearly sophisticated

This is the equilibrium configuration when $\bar{\beta}$ is low ($\bar{\beta} \in [\beta_0, \bar{\beta}_1[$) [4]. As can be seen in figure 6, this configuration is such that $\bar{\beta} < 1/R^2 = 1/(R+F)$. Thus all the agents with $\beta \leq 1/(R+F)$ (rightly) expects to rollover. Recall that the expected discount factor is $\tilde{\beta}(\beta) = \beta \geq \bar{\beta}$ for the sophisticated agents and $\tilde{\beta}(\beta) = \bar{\beta}$ for the naïve ones, which implies that the expected discount factor is small enough ($\tilde{\beta}(\beta) < 1/(R+F), \forall \beta < 1/(R+F)$) to send the right signal to the naïve borrowers (do not sign the one-period contract since you will rollover!). Therefore all agents whose $\beta > 1/(R+F)$ choose the one-period contract, whereas those with $\beta \leq 1/(R+F)$ choose the 2-period contract [5] No penalty is ever paid.

The configuration 1 equilibrium is an "honest" equilibrium where no one is fooled, and the penalty fee F is used as a commitment device for both the naïve and sophisticated borrowers. The equilibrium (R, F) pair is the same as the one chosen when all individuals are sophisticated ($R = R_S = (1/\beta_0)^{1/3}, F = F_S = R_S(R_S - 1)$).

[4]The value of $\bar{\beta}_1$ is given in appendix C

[5]The firm chooses R such that $1/R^2 = 1/(R+F)$ since $1/R^2 < 1/(R+F)$ leads to the loss of valuable consumers and $1/R^2 > 1/(R+F)$ leads to the cannibalization of 2-period contracts by the less profitable one-period contracts.

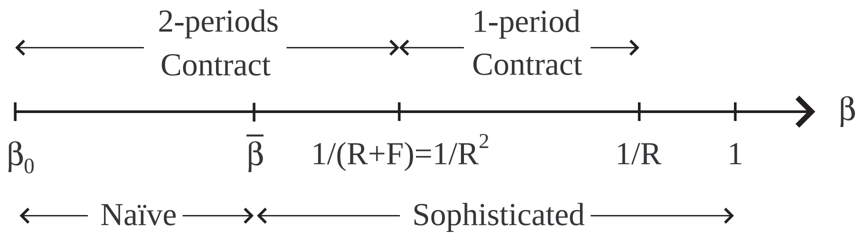

Figure 6: Configuration 1 : Nearly sophisticated

4.2.2 Configuration 2 : Sophisticated naïves

This is the equilibrium configuration for intermediate $\bar{\beta}$ ($\bar{\beta} \in [\bar{\beta}_1, \bar{\beta}_2[$) [6]. As can be seen in figure 7, this configuration is such that $\bar{\beta} = 1/(R+F) > 1/R^2$. Therefore all borrowers choose the one-period contracts (no one expects to rollover since $\tilde{\beta} \geq \bar{\beta}, \forall \beta$). This is the right decision for the sophisticated agents, since they will not rollover ($\beta \geq 1/(R+F), \forall \beta \geq \bar{\beta}$), and the wrong decision for the naïve ones (since they will rollover and pay the penalty).

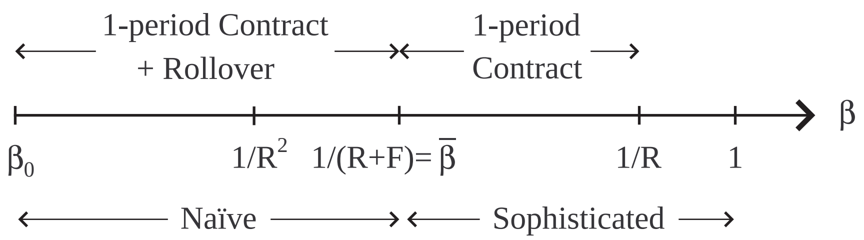

Figure 7: Configuration 2 : Sophisticated naïve

The penalty has therefore a dual role in configuration 2. The penalty is used as a commitment device for the sophisticated agents (who would choose to abstain from borrowing if $F = 0$), and as a bait for the naïve ones. Note that the optimal behavior for the naïve agents would be either to borrow for two periods (for those with $\beta < 1/R^2$) or to abstain from borrowing (for those with $\beta \geq 1/R^2$).

We show in appendix C that the optimal interest rate and penalty fee are : [7]

$$R_{SN} = \sqrt{\frac{\bar{\beta}}{\beta_0 - \bar{\beta}(1-\bar{\beta})}} \tag{9}$$

$$F_{SN} = \frac{1}{\bar{\beta}} - R_{SN} = \frac{1}{\bar{\beta}} - \sqrt{\frac{\bar{\beta}}{\beta_0 - \bar{\beta}(1-\bar{\beta}}} \tag{10}$$

The figure 8 represents the profit as a function of R when $F = F_{SN}$.

[6] The value of $\bar{\beta}_2$ is given in appendix C

[7] The subscript $_{SN}$ stands for sophisticated naïve.

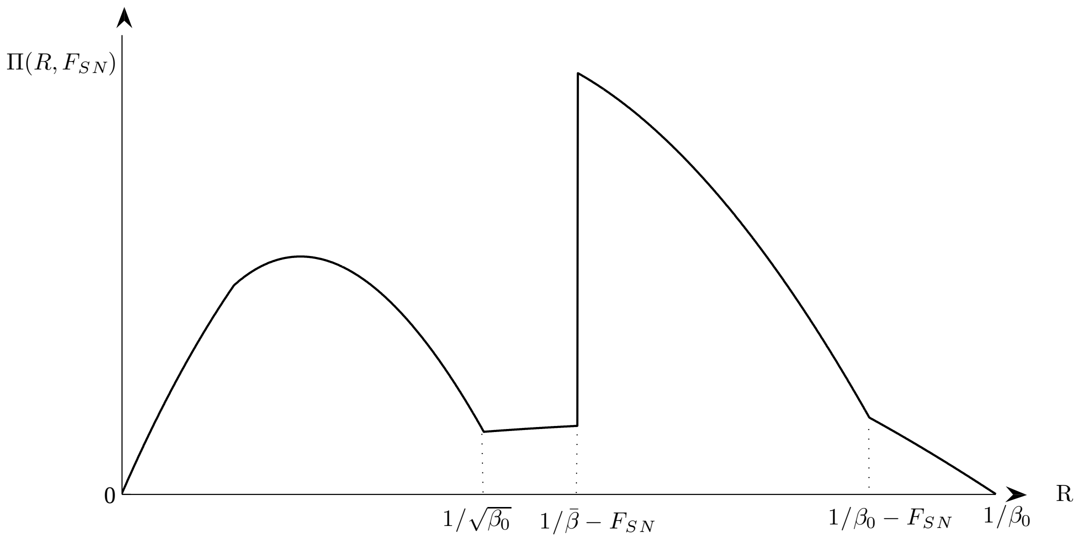

Figure 8: The profit function with sophisticated naïves.

4.2.3 Configuration 3 : Nearly naïve

This is the equilibrium configuration when $\bar{\beta}$ is high ($\bar{\beta} \in [\bar{\beta}_2, 1]$). As can be seen in figure 9, this configuration is such that $\bar{\beta} \geq 1/R$. The optimal interest rate is such that : (i) sophisticated agents do not borrow ; and (ii) all naïve agents borrow for one period and then rollover in the second period. As in fully naive case, the choice of the optimal penalty fee must balance the additional income generated by the fee and the loss of some two-period contracts caused by this fee. We show in appendix C that, as in the fully naïve case the optimal penalty fee is zero. [8].

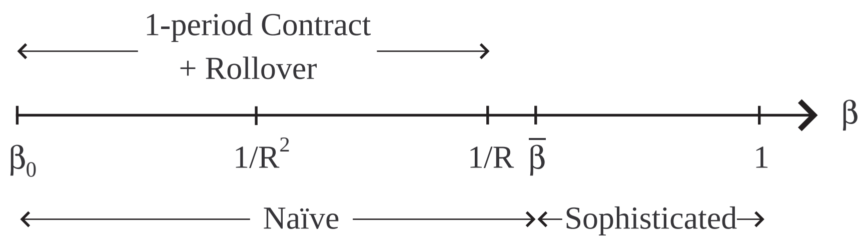

Figure 9: Configuration 3 : Nearly naïve

There are two sub-configurations, depending on the value of $\bar{\beta}$. Configuration 3.1, which is the equilibrium configuration for $\bar{\beta} \in [\bar{\beta}_2, 1/R_N[$, is such that :[9]

$$R_{NN} = 1/\bar{\beta} \tag{11}$$

$$F_{NN} = 0 \tag{12}$$

[8]Note that the borrowers can be considered as fully naïve (even if they have on average an expected $\tilde{\beta}$ closer to their true β, they act exactly as if they were fully naïves in equilibrium)

[9]The subscript $_{NN}$ stands for nearly naïve.

Configuration 3.2, which is the equilibrium configuration for $\bar{\beta} \in [1/R_N, 1]$, is such that :

$$R_{NN} = R_N \tag{13}$$
$$F_{NN} = 0 \tag{14}$$

where R_N is the interest rate chosen by the firm when consumers are fully naïve (section section 4.1).

4.3 The impact of naïveté on the optimal strategies and the profit per customer

The figures 10, 11 and 12 plot the optimal interest rate, the optimal fee and the profit per customer as a function of the index of naïveté $\bar{\beta}$ when $\beta_0 = 2/2$.

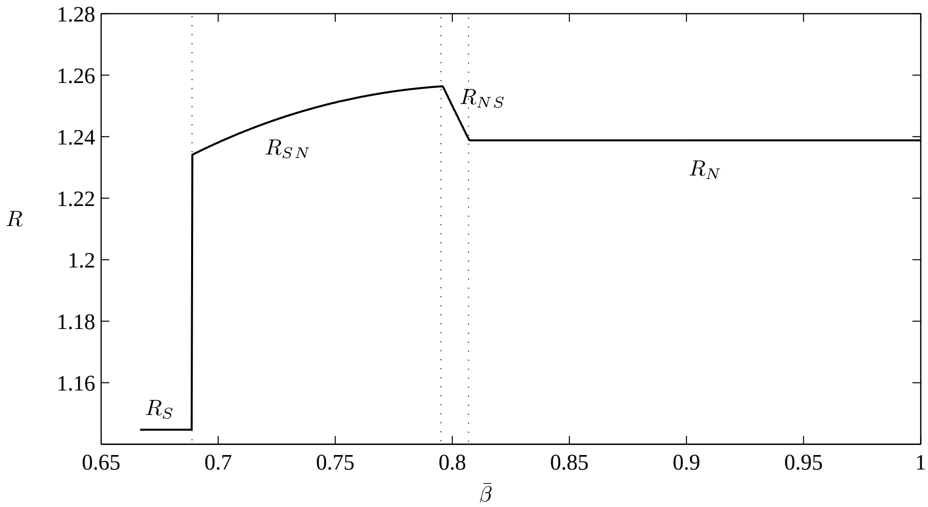

Figure 10: The optimal interest rate as a function of the index of naïveté $\bar{\beta}$

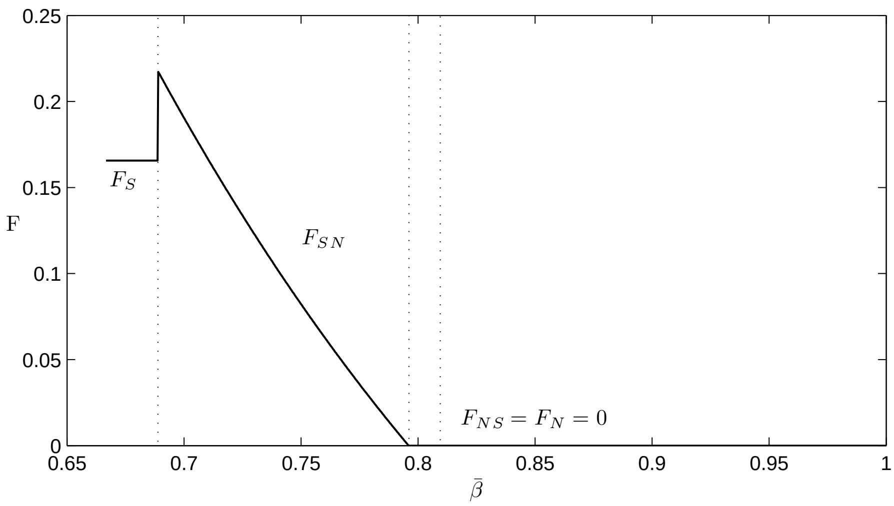

Figure 11: The optimal fee as a function of the index of naïveté $\bar{\beta}$

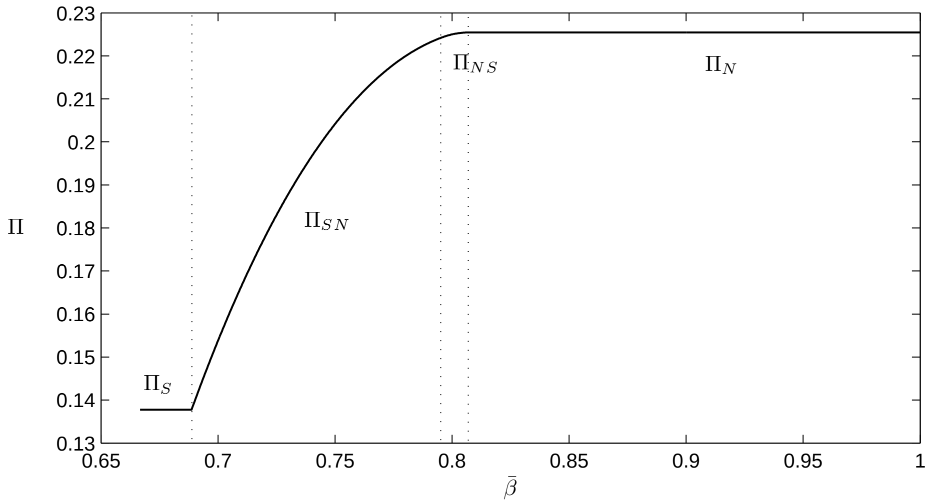

Figure 12: The profit per customer as a function of the index of naïveté $\bar{\beta}$

5 The Impact of Competition

This section studies the impact of full competition on the equilibrium pricing strategies. We assume the following : (i) entry is unrestricted and costless, (ii) there are no capacity constraints, and (ii) firms compete by announcing a $(R,\ F)$ couple. We show that the impact of competition depends crucially on the degree of naïveté of the borrowers. We first study the polar cases (full naïveté and full sophistication), then we solve the intermediate case ($\beta_0 < \bar{\beta} < 1$).

5.1 Fully sophisticated versus fully naïves

Lets's examine the impact of competition when the borrowers are fully sophisticated. Competition is a two dimensions game, since firms may increase their market share either by reducing R or by increasing F. However, it is easy to show that choosing a slightly lower interest rate than the other firms is always the dominant strategy. Suppose for instance that there are two identical firms and let $R_2 > 1$ and $F_2 > 0$ be the interest rate and the fee chosen by firm 2. Firm 1 may increase its profit by choosing any $F_1 > F_2$, as long as $(1/R_2)^2 < 1/(R_2 + F_2)$ (Firm 1 takes all the consumers in the interval $[1/(R_2 + F_1), 1/(R_2 + F_2)]$, which yields a net profit equal to $(R - 1)$ per consumer) [10]. But decreasing marginally the interest rate (choosing $R_1 = R_2 - \epsilon$) is a much better strategy, since it allows the firm to take the whole market (profits are multiplied by a factor slightly less than 2). Of course, the same logic applies for firm 2. Therefore the equilibrium is such that $R_1 = R_2 = 1$, which implies that the fees are no longer needed. The same reasoning applies for any number of firms. Firm's profits are null at equilibrium and all the surplus is taken by the consumers. Furthermore, time inconsistency is no longer a problem, since borrowing at period 2 at interest rate $R = 1$ entails no loss from the point of view of the period one self.

Has competition the same beneficial effects when borrowers are fully naïve? Naïve borrowers are not aware of the benefits and dangers of the penalty fees, since they don't

[10] This applies as long as $1/(R_2 + F_1) \geq (1/R_2)^2$. Increasing further R_1 leads to a loss for firm 1, due to the cannibalization of the 2-period contracts by the 1-period ones.

realize their time inconsistency problem. They, therefore, do not take into account the fee when comparing the borrowing contracts. The only relevant variable in the (R, F) contract is the the interest rate, which implies that the firms are free to set the fee at the level they see fit. This open the way for a new strategy based on deceit.

Suppose, for instance, that all the other firms set $R = 1$ (the equilibrium level when agents are sophisticated). A positive profit can be made with the following strategy : offer a one period contract with a "discount" interest rate $R < 1$ and the possibility to rollover at cost $R + F$ [11]. All agents will take the contract, and then a fraction $G(1/(R + F))$ will rollover (those whose preferences are such that $\beta \leq 1/(R + F)$). The firm will loose $R - 1$ per contract on the one period borrowers (whose mass is $1 - G(1/(R + F))$ and gain $R(R + F) - 1$ per contract on the $G(1/(R + F))$ individuals who will rollover. The expected profit of the firm thus writes :

$$\Pi(R, F) = (R(R + F) - 1)G\left(\frac{1}{R + F}\right) + (R - 1)\left(1 - G\left(\frac{1}{R + F}\right)\right) \tag{15}$$

which is positive for some $R < 1$ and $F > 0$ (see the end of appendix D.1). Of course, all other firms will offer the same contract. Then free entry implies, again, that the equilibrium will be a zero-profit one. A symmetric equilibrium will therefore be characterized by the following three conditions : (i) all firms choose the same interest rate R ; (ii) each firm chooses the penalty fee F so as to maximize her expected profit given R ; (iii) expected profits are null. In the appendix D.1, we show that these three conditions imply that the equilibrium (R, F) are :

$$R_N^c = \frac{1 + \sqrt{\beta_0}}{2} \tag{16}$$

$$F_N^c = \frac{1}{\sqrt{\beta_0}} - \frac{1 + \sqrt{\beta_0}}{2} \tag{17}$$

where the superscript ^{c} stands for competition, and the subscript $_N$ for fully naïve.

5.2 The equilibrium with partially naïve agents

Recall that the agent's population is divided in two groups : the sophisticated ones (those with $\beta \geq \bar{\beta}$) and the partially naïves (those with $\beta < \bar{\beta}$). Both groups are aware of their time inconsistency problem and, therefore, do realize the commitment value of the fee. The big difference between the two groups is that the partially naïve can be fooled by an appropriate contracts, whereas the sophisticated cannot.

The deceitful contract has the same structure as before : a one period lending contract with a "discount" interest rate $R < 1$, and the possibility to rollover at cost $R + F$. The key condition is that R and F must verify :

$$\bar{\beta} \geq \frac{1}{R + F} \tag{18}$$

Condition (18) implies that nobody expect to roll-over, since the expected discount factor is $\tilde{\beta}(\beta) \geq \bar{\beta} \geq 1/(R + F)$. When the firms offer this kind of contract, the game is very much the same as the one played with fully naïve agents. The firms are free to set the

[11] The firm may also propose a portfolio of two contracts, where contract 1 is the standard contract at rate $R = 1$ (with costless rollover) and contract 2 is the discount contract ($R < 1$), with a penalty F in case of rollover. The outcome will be the same, since the expected surplus from contract 1 ($1 - \beta$) is lower than the expected surplus for contract 2 ($1 - \beta R$) for all naïves.

level of the fee, since all agents consider that the fee is big enough to deter rollover (and nobody expects to pay the fee). Therefore firms compete by offering the lowest interest rate. Free entry implies a zero-profit equilibrium.

A symmetric equilibrium will therefore be characterized by the following three conditions : (i) all firms choose the same interest rate R ; (ii) each firm chooses the penalty fee F so as to maximize her expected profit given R subject to condition (18); (iii) expected profits are null. It is easy to show (see appendix D.2) that there are two equilibrium configuration.

The constrained equilibrium arises when the share of partially naïve agents is low ($\bar{\beta} < \sqrt{\beta_0}$). F is such that all the partially naïve rollover $R + F = 1/\bar{\beta}$. Then, the zero profit condition yields :

$$R^c_{PN} = \frac{\bar{\beta}(1-\beta_0)}{\bar{\beta}(1-\beta_0) + (1-\bar{\beta})(\bar{\beta}-\beta_0)} \tag{19}$$

$$F^c_{PN} = \frac{1}{\bar{\beta}} - \frac{\bar{\beta}(1-\beta_0)}{\bar{\beta}(1-\beta_0) + (1-\bar{\beta})(\bar{\beta}-\beta_0)} \tag{20}$$

where the subscript $_{PN}$ stands for partially naïve.

The unconstrained equilibrium arises when the share of partially naïve agents is high ($\bar{\beta} \geq \sqrt{\beta_0}$). The firms make the same choice as in the fully naïve case. Therefore :

$$R^c_{PN} = R^c_N \tag{21}$$

$$F^c_{PN} = \frac{1}{\sqrt{\beta_0}} - R^c_N \equiv F^c_N \tag{22}$$

and $R^c_{PN} + F^c_{PN} = 1/\sqrt{\beta_0} > 1/\bar{\beta}$. Figures (13) and (14) show the equilibrium F and R as a function of the index of naïveté $\bar{\beta}$. Both the fee and the interest rate decrease with $\bar{\beta}$ (this is a general result, $\partial R^c_{PN}/\partial\bar{\beta} < 0$ and $\partial F^c_{PN}/\partial\bar{\beta} < 0$, $\forall \bar{\beta} \in]\beta_0, 1]$). Incumbent firms react to the increase of the share of the naïves $G(\bar{\beta})$, by cutting the penalty fee F, which reduces the profit par naïve customer but increases the market share of the exploitative contracts. The latter effect dominates as long as $1/(R+F) \leq \sqrt{\beta_0}$ (recall that $R + F = 1/\sqrt{\beta_0}$ maximizes the expected profit for fully naïve borrowers). Positive profits lead to the entry of new firms, which brings down the interest rate.

Note that the pricing policy engenders a cross-subsidy from the naïves to the sophisticated individuals (and some of the naïves too when $\bar{\beta} \geq \sqrt{\beta_0}$). Each one of the $G(1/(R+F))$ naïves that roll-over "pays" $R(R+F)-1$ to the $1-G(1/(R+F))$ individuals that manage to stick to the one period loan. Note also that, somewhat counterintuitively, the intensity of exploitation (as measured by the penalty fee) is globally decreasing with the index of naïveté, and is maximal when the share of naïves id close to zero.

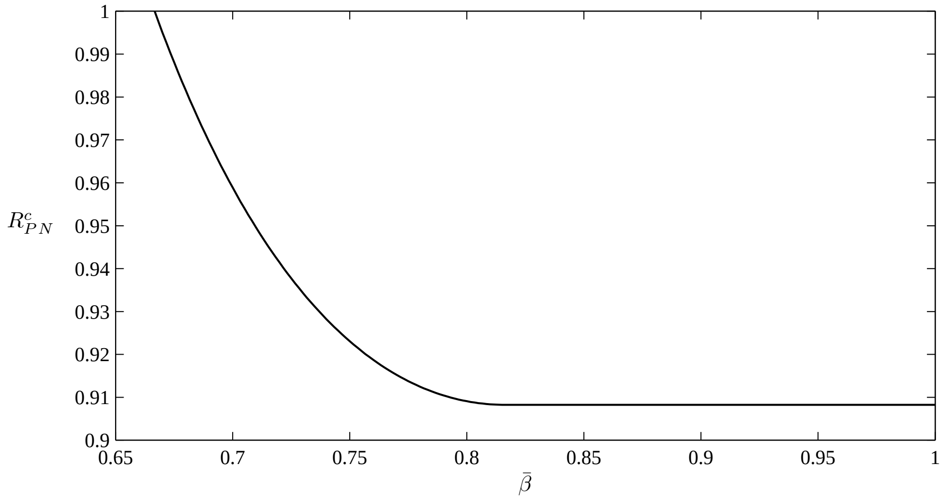

Figure 13: The equilibrium interest rate under competition as a function of the index of naïveté $\bar{\beta}$

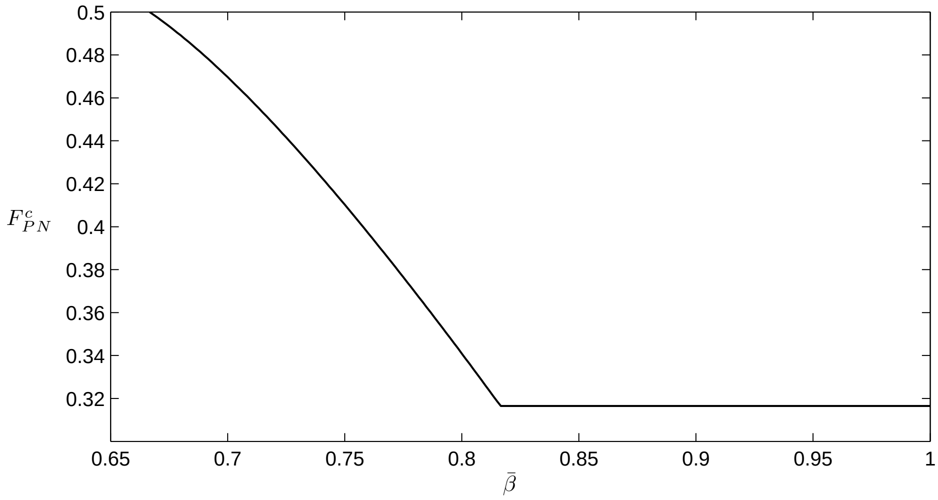

Figure 14: The equilibrium fee under competition as a function of the index of naïveté $\bar{\beta}$

6 A Welfare Analysis

6.1 A paternalistic approach

Evaluating the borrower's welfare is difficult since preferences are time-varying. One solution, proposed by O'Donoghue and Rabin (1999a,b), is to distinguish between the long-run welfare and the actual welfare. Note that the intertemporal utility of the agent can be rewritten as :

$$u_t + \beta \sum_{s \geq t+1} \delta^{s-t} u_s = (1-\beta) u_t + \beta V_t^{LR} \tag{23}$$

where $V_t^{LR} = \sum_{s \geq t} \delta^{s-t} u_s$ stands as the long run utility of the agent. With this formulation the actual intertemporal utility of the individual is an average of his short-run utility

(u_t) and his long-run utility (V_t^{LR}). Rabin asserts that V_t^{LR}, which is time-consistent, is the relevant measure of the agent's welfare.

With V_t^{LR} as the welfare index any type of borrowing is harmful in a monopoly setting since $R > 1$ (recall that $\delta = 1$ and u_t is linear). In this sense all contracts that lead some agents to borrow either for one period or two periods are exploitative in a monopoly setting. The aggregate welfare losses are just equal to the monopoly profits, which are an increasing function of the index of naïveté $\bar{\beta}$ (this is due to the fact that the net global surplus is equal to zero). However the individual losses depend on a non trivial way on the borrower's impatience β and naïveté $\tilde{\beta}(\beta)$. When all the individuals are sophisticated, the more impatient (those who take a two-period loan) loose $1 - (R_S)^2$ each, whereas the patient borrowers (those who take a one-period loan) loose $1 - R_S$ each [12]. When all the individuals are naïves, all those who borrow take a two-period loan which leads to a loss equal to $1-(R_N)^2$. This loss is clearly greater that the losses incurred by any sophisticated borrower, weather patient or impatient, since $R_N > R_S$. Yet the partially naïves may loose more than the fully naïves. The patient ones (those who borrow for one period) pay a higher interest rate ($R_{SN} > R_N$), and the impatient ones end up paying a two-period loan plus a penalty fee, which leads to a loss equal to $1 - R_{SN}(R_{SN} + F_{SN}) < 1 - (R_N)^2$ (her loss is greater than the loss sustained by a fully naïve that take a two-period loan). Partial naïveté is therefore worse than full naïveté for those who borrow.

Perfect competition drives the profits to zero. This has a positive impact on the average borrower's welfare, which is now equal to zero (recall that the net global surplus is zero). However the impact of competition on individuals crucially depends on their naïveté. Sophisticated individuals (those who manage to borrow for one period) can't loose from competition. When everybody is sophisticated ($\bar{\beta} = \beta_0$), the interest rate is $R = 1$, which implies that all individuals have a surplus equal to zero. When some of the individuals are naïve, the sophisticated earn a positive surplus since $R < 1$. Note that this surplus rises with the index of naïveté $\bar{\beta}$ (since R is a decreasing function of $\bar{\beta}$). The gains of the sophisticated are the losses of the naïves (those who borrow for two periods and therefore pay the penalty fee). It must be noted that the losses of the naïves $1-R(R+F)$ are, generally, lower under competition. Let's show this when all individuals are fully naïve. Under monopoly, all individuals in the $[\beta_0, 1/R_N]$ interval borrow for one period and then rollover, which lead to a loss equal to $1-(R_N)^2$ (there is no penalty fee). Under perfect competition, the same applies for all the individuals in the $[\beta_0, \sqrt{\beta_0}]$ interval, which lead to a loss equal to $1 - R_N^c(R_N^c + F_N^c)$. The welfare loss under competition is lower since $R_N^c(R_N^c + F_N^c) = R_N^c/\sqrt{\beta_0} < 1/\sqrt{\beta_0} < R_N < (R_N)^2$. Yet some naïves are made worse off by competition : those who do not borrow under monopoly (those whose $\beta \in]1/R_N, \sqrt{\beta_0}]$). Figure 15 show the extent of two period borrowing under monopoly and perfect competition.

[12] Note that there is always a fraction a of the potential borrowers (the very patient) that choose to abstain from borrowing.

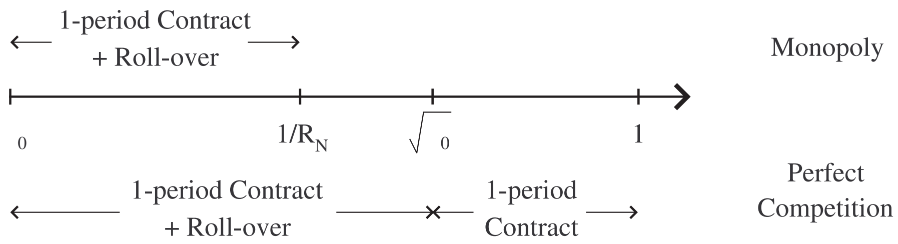

Figure 15: Exploitative contracts under monopoly and perfect competition with fully naïve agents

The same logic applies under partial naïveté. The naïves that roll-over under monopoly and competition benefit from competition, since they pay lower rates (including the penalty fee). But the naïves who do not borrow under monopoly are made worse-off by competition. [13]

6.2 Welfare from the point of view of self one

Using the long run utility as a measure of welfare has one big drawback : it may lead to violate the individual's choice at each point of time. This certainly can be considered as hard paternalism. Sunstein and Thaler (2003) preconize a softer approach, which they call libertarian paternalism[14]. The purpose of libertarian paternalism is to help the individual to make freely the "right" choices. This implies that another welfare measure must be chosen. Utility at time one seems to be the natural choice, since it represents the wills of at least one of the multiple selves of the individual and is not too far from the long-run utility [15]. Note that the credit market is no longer useless, as it fulfills the "legitimate" need for credit of self one. Lending for one period creates a surplus equal to $1-\delta$, which is shared between the bank and the borrower.

Sophisticated agents cannot loose from borrowing. Their need for commitment devices is partially fulfilled under monopoly (they would prefer a higher penalty fee), and entirely fulfilled under competition (since the optimal contract deters them from renewing the loan). Competition clearly benefits them since they take the whole surplus of the exchange (plus a transfer from the naïves).

Naïves may loose from borrowing under monopoly, since they over-estimate the benefits of the loan. For instance when all the borrowers are fully naïve, of all the individuals who borrow and then roll-over the loan, all those whose $\beta \in](1/R_N)^2, 1/R_N]$ have a welfare loss. This is no longer the case under perfect competition. The individuals that roll-over the loan have a welfare gain equal to $1-\beta R_N^c(R_N^c+F_N^c) = 1-\beta R_N^c/\sqrt{\beta_0} > 0$ (since $\beta < \sqrt{\beta_0}$). In fact all naïves are made better off by competition, even those who do not

[13]There are in fact two cases. When the index of naïveté is low ($\bar{\beta} \leq \sqrt{\beta_0}$), the same population borrows and roll-over under monopoly and competition (those whose $\beta \in [\beta_0, \bar{\beta}]$). Therefore nobody looses from competition. When the index of naïveté is high ($\bar{\beta} > \sqrt{\beta_0}$), a fraction of the individuals that borrow and roll-over under competition do not so under monopoly (those whose $\beta \in]1/R_N, \sqrt{\beta_0}]$). This is the population that looses from competition.

[14]Camerer et al. (2003) use the term "asymmetric paternalism", others prefer the slightly ironic "soft paternalism".

[15]We mean, of course, self one real utility (the one he will derive by his effective consumption path), as opposed to his expected utility.

roll-over under monopoly[16]. These results extend to the partially naïve case[17].

7 Conclusion

This paper studies the equilibrium predatory practices that may arise when the borrowers have behavioral weaknesses. Rational lenders offer short term contracts that can be renewed at the cost of paying a penalty fee. We show how the optimal contracts depend on the degree of naïveté of the time inconsistent customers. Penalty fees have a dual role : they increase market share by providing a useful commitment device to time-inconsistent but otherwise rational borrowers ; they are also a source of revenue from the semi-naïve borrowers who understand the need for commitment but fail to forecast their future time discount factor. We also show that perfect competition does not eliminate predatory practices, since the equilibrium contract entails a subsidized (below marginal cost) short-term loan that can only be profitable if a fraction of the borrowers end up paying the penalty fee.

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[16]Proof. The welfare gain for the fully naïve is, under monopoly : $1-\beta(R_N)^2$ if $\beta < 1/R_N$ and 0 if $\beta \geq 1/R_N$; and under perfect competition $1-\beta R_N^c/\sqrt{\beta_0}$ if $\beta < \sqrt{\beta_0}$, and $1-\beta R_N^c$ if $\beta \geq \sqrt{\beta_0}$. $1/R_N < \sqrt{\beta_0}$, therefore we have three cases: (i) $\beta \in [\beta_0, 1/R_N[$, $1-\beta(R_N)^2 < 1-\beta R_N^c/\sqrt{\beta_0}$ since $R_N > 1/\sqrt{\beta_0} > 1 > R_N^c$; (ii) $\beta \in [1/R_N, \sqrt{\beta_0}[$,$0 < 1-\beta R_N^c/\sqrt{\beta_0}$, since $\beta < \sqrt{\beta_0}$ and $R_N^c < 1$; (iii) $\beta \in [\sqrt{\beta_0}, 1]$,$0 < 1-\beta R_N^c$, since $R_N^c < 1$.

[17]Proof. If $\bar{\beta} \geq \sqrt{\beta_0}$...

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A Sophisticated borrowers

A.1 The profit function when $F \geq \frac{1}{\beta_0} - \sqrt{\frac{1}{\beta_0}}$

In this case, the profit function writes :

$$\Pi(R,F) = \begin{cases} (R^2-1)G\left(\frac{1}{R+F}\right) + (R-1)\left[G\left(\frac{1}{R}\right) - G\left(\frac{1}{R+F}\right)\right] & \text{if} \quad 1 \leq R < \frac{1}{\beta_0} - F \\ (R-1)G\left(\frac{1}{R}\right) & \text{if} \quad \frac{1}{\beta_0} - F \leq R \leq \frac{1}{\beta_0} \end{cases} \tag{24}$$

Proof. All individuals choose one-period contract when $1/\beta_0 - F \leq R \leq 1/\beta_0$, since they are profitable ($\beta R < 1$, $\forall \beta$) and the individuals know that they will not rollover ($\beta(R+F) > 1$, $\forall\beta$). The profit is therefore $(R-1)$ times the number of contracts $G(1/R)$. $R < 1/\beta_0 - F < 1/\sqrt{\beta_0}$ imply that $\beta_0 < 1/(R+F) < 1/R^2 < 1/R$ (recall that $F \geq 1/\beta_0 - \sqrt{1/\beta_0} \Longrightarrow R_F \geq \sqrt{\beta_0}$). From this we know that : (i) some individuals will roll-over (those such that $\beta < 1/(R+F)$); (ii) 2-period contracts are profitable for those individuals (since $1/(R+F) < 1/R^2$). All the other individuals choose one-period contracts. The profit is therefore (R^2-1) times the number of 2-period contracts $G(1/(R+F))$ plus $(R-1)$ times the number of one-period contracts $G(1/R) - G(1/(R+F))$.

Let's show that the optimal F cannot be greater than $\frac{1}{\beta_0} - \sqrt{\frac{1}{\beta_0}}$.

First we show that the interest rate R^* that maximizes $\Pi(R,F)$ (for a given $F \geq 1/\beta_0 - \sqrt{1/\beta_0}$) lies in the interval $]1, 1/\sqrt{\beta_0}]$. Let's assume that $R^* > 1/\beta_0 - F$. The profit function then writes $\Pi(R,F) = (R-1)G(1/R) = \Pi_1(R)$. The first and second derivative of $\Pi_1(R)$ are :

$$\begin{aligned} \Pi_1'(R) &= \frac{1}{\beta_0}\left(\frac{1}{R} - \beta_0 - 1 + \frac{1}{R^2}\right) \\ \Pi_1''(R) &= -\frac{1}{\beta_0}\left(\frac{1}{R^2} + \frac{2}{R^3}\right) \end{aligned}$$

$\Pi_1''(R) < 0$ implies that $\Pi_1(R)$ has a unique maximum and $\Pi_1'(1/\sqrt{\beta_0}) = \frac{\sqrt{\beta_0}-1}{\beta_0-1} < 0 \Longrightarrow R^* < 1/\sqrt{\beta_0}$.

$R^* < 1/\sqrt{\beta_0}$ implies that $1/(R^*+F) < (1/R^*)^2$ (since $R_F > 1/\sqrt{\beta_0}$). Such a configuration can't be optimal since the profit can be increased by *reducing* F (which allows the firm to replace one-period contracts by the 2-period ones). This shows that the optimal F is such that $F < 1/\beta_0 - \sqrt{1/\beta_0}$.

A.2 The optimum

Taking into account that the optimal F is such that $F = R_F(R_F-1)$, we can write the profit function as :

$$\begin{aligned}
\Pi_S(F) &= \left(R_F^2-1\right) G\left(\frac{1}{R_F^2}\right) + (R_F-1)\left(G\left(\frac{1}{R_F}\right) - G\left(\frac{1}{R_F+F}\right)\right) \\
&= R_F\left(R_F-1\right) G\left(\frac{1}{R_F^2}\right) + (R_F-1)\, G\left(\frac{1}{R_F}\right) \\
&= \frac{1}{1-\beta_0}\left(F \times \left(\frac{1}{R_F^2} - \beta_0\right) + (R_F-1)\left(\frac{1}{R_F} - \beta_0\right)\right)
\end{aligned}$$

Using $R_F' = \frac{dR_F}{dF} = \frac{1}{2R_F-1}$ and $F = R_F(R_F-1)$ we compute the first and second derivatives :

$$\begin{aligned}
\Pi_S'(F) &= \frac{1}{1-\beta_0}\left(\frac{1}{R_F^2} - \beta_0 - 2F\frac{R_F'}{R_F^3} + \frac{R_F'}{R_F^2} - \beta_0 R_F'\right) \\
&= \frac{2}{1-\beta_0} \times \frac{R_F^2 - \beta_0 R_F^4 - F}{R_F^3(2R_F-1)} \\
&= \frac{2}{1-\beta_0} \times \frac{1 - \beta_0 R_F^3}{R_F^2(2R_F-1)} \\
\Pi_S''(F) &= \frac{2R_F'}{1-\beta_0} \times \frac{\beta_0 R_F^3 - 6R_F + 2}{R_F^3(2R_F-1)^2} \\
&= -\frac{2}{1-\beta_0} \times \frac{3}{R_F^3(2R_F-1)^2} < 0
\end{aligned}$$

Since $\Pi_S''(F) < 0$ the maximum of the function Π_S is attained when $\Pi_S'(F) = 0 \Leftrightarrow R_F = (1/\beta_0)^{1/3}$.

B Fully naïve borrowers

Since $F_N = 0$, R_N is the maximum of the function :

$$\Pi_N = \Pi(R,0) = (R^2-1)G(1/R) = \frac{(R^2-1)(1/R-\beta_0)}{1-\beta_0}$$

in the interval $[1, 1/\beta_0]$. The function $\Pi_N(R)$ is concave (since $\Pi_N''(R) = -2\frac{\beta_0+1/R^3}{1-\beta_0}$). The maximum is the solution of

$$\Pi_N'(R) = \frac{1}{1-\beta_0}\left(1 - 2\beta_0 R + \frac{1}{R^2}\right) = 0$$

which yields a cubic equation ($-2\beta_0 R^3 + R^2 + 1 = 0$). Since $\Pi_N'(1) = 2 > 0$ and $\Pi_N'(1/\beta_0) = -(1+\beta_0) < 0$, this equation has a unique solution in the interval $[1, 1/\beta_0]$.

$\Pi_N'(1/\sqrt{\beta_0}) = (1-\sqrt{\beta_0})^2/(1-\beta_0) > 0$ implies that $1/\sqrt{\beta_0} < R_N < 1/\beta_0$.

C Partially Naïve borrowers

C.1 The profit function

For the sake of simplicity we write the profit function as a function of R and $Z = R+F$. There are five different regions (see figures 16 and 17 for a mapping in the (R, Z) space). Hereupon we show that the profit function writes, when $\beta_0 < \bar{\beta} < \sqrt{\beta_0}$ (figure 16 and 17)

$$\Pi(R,Z) = \begin{cases} \Pi_1(R,Z) \equiv (R-1)G\left(\frac{1}{R}\right) & \text{if} \quad Z \geq \frac{1}{\beta_0} \quad \text{and} \quad R \leq Z \\ \Pi_2(R,Z) \equiv R(Z-1)G\left(\frac{1}{Z}\right) + (R-1)G\left(\frac{1}{R}\right) & \text{if} \quad \frac{1}{\bar{\beta}} \leq Z < \frac{1}{\beta_0} \quad \text{and} \quad R \leq Z \\ \Pi_3(R,Z) \equiv (R-1)\left[G\left(\frac{1}{R}\right) - G\left(\frac{1}{Z}\right)\right] & \text{if} \quad Z < \frac{1}{\bar{\beta}} \quad \text{and} \quad \frac{1}{\sqrt{\beta_0}} \leq R \leq Z \\ \Pi_4(R,Z) \equiv (R^2-1)G\left(\frac{1}{R^2}\right) + (R-1)\left[G\left(\frac{1}{R}\right) - G\left(\frac{1}{Z}\right)\right] & \text{if} \quad Z < \frac{1}{\bar{\beta}} \quad \text{and} \quad \sqrt{Z} \leq R < \max\left\{Z, \frac{1}{\sqrt{\beta_0}}\right\} \\ \Pi_5(R,Z) \equiv R(R-1)G\left(\frac{1}{Z}\right) + (R-1)G\left(\frac{1}{R}\right) & \text{if} \quad Z < \frac{1}{\bar{\beta}} \quad \text{and} \quad R < \sqrt{Z} \end{cases} \tag{25}$$

and, for $\beta_0 < \sqrt{\beta_0} \leq \bar{\beta}$ (figure 18):

$$\Pi(R,Z) = \begin{cases} \Pi_1(R,Z) \equiv (R-1)G\left(\frac{1}{R}\right) & \text{if} \quad Z \geq \frac{1}{\beta_0} \quad \text{and} \quad R \leq Z \\ \Pi_2(R,Z) \equiv R(Z-1)G\left(\frac{1}{Z}\right) + (R-1)G\left(\frac{1}{R}\right) & \text{if} \quad \frac{1}{\bar{\beta}} \leq Z < \frac{1}{\beta_0} \quad \text{and} \quad R \leq Z \\ \Pi_4(R,Z) \equiv (R^2-1)G\left(\frac{1}{R^2}\right) + (R-1)\left[G\left(\frac{1}{R}\right) - G\left(\frac{1}{Z}\right)\right] & \text{if} \quad Z < \frac{1}{\bar{\beta}} \quad \text{and} \quad \sqrt{Z} \leq R \leq Z \\ \Pi_5(R,Z) \equiv R(R-1)G\left(\frac{1}{Z}\right) + (R-1)G\left(\frac{1}{R}\right) & \text{if} \quad Z < \frac{1}{\bar{\beta}} \quad \text{and} \quad R < \sqrt{Z} \end{cases} \tag{26}$$

Proof:

Region 1: $Z \geq \frac{1}{\beta_0}$ and $R \leq Z \Longrightarrow 1/Z \leq \beta_0 < \bar{\beta} < 1/R$. All individuals choose the one period contract, since they don't expect to rollover ($\tilde{\beta}(\beta) \geq \bar{\beta} > 1/Z, \forall \beta$). No one rollovers since $\beta \geq \beta_0 > 1/Z$. The profit is $(R-1)$ times the number of contracts $G(1/R)$.

Region 2: $\frac{1}{\bar{\beta}} \leq Z < \frac{1}{\beta_0}$ and $R \leq Z \Longrightarrow \beta_0 < 1/Z \leq \bar{\beta} < 1/R$. As before all individuals choose the one period contract. But some individuals will rollover (those with $\beta < 1/Z$). The profit is $(R-1)$ times the number of one period contracts $G(1/R) - G(1/Z)$ plus $RZ - 1$ times the number of individual who rollover $G(1/Z)$.

Region 3: $Z < \frac{1}{\bar{\beta}}$ and $\frac{1}{\sqrt{\beta_0}} \leq R \leq Z \Longrightarrow 1/R^2 \leq \beta_0 < \bar{\beta} < 1/Z < 1/R$. All individuals with $\beta > 1/Z$ choose (rightly) the one period contracts since $\tilde{\beta}(\beta) = \beta > 1/Z, \forall \beta > 1/Z$. All the other individuals expect to rollover, and thus abstain from borrowing (two-period contracts are not profitable since $\beta > 1/R^2, \forall \beta$). The profit is $(R-1)$ times the number of one period contracts $G(1/R) - G(1/Z)$.

Region 4: $Z < \frac{1}{\bar{\beta}}$ and $\sqrt{Z} < R < \max\left\{Z, \frac{1}{\sqrt{\beta_0}}\right\} \Longrightarrow \beta_0 < \max\{\bar{\beta}, 1/R^2\} \leq 1/Z < 1/R$. Same logic as in the previous case. All the individuals with $\beta < 1/Z$ rightly expects to rollover. They either abstain from borrowing or choose a two period contract (which is profitable for those with $\beta < 1/R^2$). The profit is $(R-1)$ times the number of one period contracts $G(1/R) - G(1/Z)$ plus $R^2 - 1$ times the number of two period contracts $G(1/R^2)$.

Region 5: $Z < \frac{1}{\bar{\beta}}$ and $R \leq \sqrt{Z} \Longrightarrow \beta_0 < \bar{\beta} < 1/Z < 1/R^2 < 1/R$. Same logic as in the previous case. All the individuals with $\beta < 1/Z$ choose a two period contract. The profit is $(R-1)$ times the number of one period contracts $G(1/R) - G(1/Z)$ plus $R^2 - 1$ times the number of two period contracts $G(1/Z)$.

C.2 The profit maximizing strategy when the borrowers are partially naïve

It is easy to see that the profit function is not convex, and has several discontinuities (the profit jumps between region 2,3 and 4 and between region 2 and region 5). Our method is first to compute the sign of the partial derivatives in each region in order to locate the local maxima, and then compare those maxima. The arrows in figures 16, 17 and 18 show the "directions" that the firm must follow to increase her profit.

We distinguish three cases.

Case 1: $\beta_0 \leq \bar{\beta} < \sqrt{\beta_0}$ **and** $R_{SN} = \sqrt{\frac{\bar{\beta}}{\beta_0 - \bar{\beta}(1-\bar{\beta})}} < \frac{1}{\bar{\beta}}$

The optimum profit strategy are :

$$\begin{cases} \text{(S)} \quad R = \left(\frac{1}{\beta_0}\right)^{1/3} \equiv R_S \quad \text{and} \quad Z = \left(\frac{1}{\beta_0}\right)^{2/3} & \text{if} \quad \beta_0 \leq \bar{\beta} < \bar{\beta}_1 \\ \text{(SN)} \quad R = \sqrt{\frac{\bar{\beta}}{\beta_0 - \bar{\beta}(1-\bar{\beta})}} \equiv R_{SN} \quad \text{and} \quad Z = 1/\bar{\beta} & \text{if} \quad \bar{\beta}_1 \leq \bar{\beta} < \bar{\beta}_2 \end{cases}$$

where $\bar{\beta}_1 < \bar{\beta}_2 < 1$ are given at the end of the subsection.

Proof.

First we show that the optimal strategy in this configuration must be either (S) or (SN) (see figure 16). Let's begin with region 1. The profit function does not depends on Z. The derivative with respect to R is :

$$(1-\beta_0)\,\frac{\partial \Pi_1(R,Z)}{\partial R} = \frac{1}{R^2} - \beta_0 \left(\lesseqgtr\right) 0 \quad \text{if} \quad R \left(\gtrless\right) \frac{1}{\sqrt{\beta_0}}$$

The local maximum is therefore attained when $R = 1/\sqrt{\beta_0}$ $(\forall Z \geq 1/\beta_0)$. But the global optimum cannot lie in region 1 since $\Pi_2(R,Z) - \Pi_1(R,Z) = (RZ-1)G(1/Z) > 0\,\forall Z > 1$.

Let's move to region 2. The partial derivatives of the profit function are :

$$\begin{aligned} (1-\beta_0)\,\frac{\partial \Pi_2(R,Z)}{\partial Z} &= R\left(\frac{1}{Z^2} - \beta_0\right) < 0 \quad \text{since} \quad Z > \frac{1}{\bar{\beta}} > \frac{1}{\sqrt{\beta_0}} \\ (1-\beta_0)\,\frac{\partial \Pi_2(R,Z)}{\partial R} &= (Z-1)\left(\frac{1}{Z} - \beta_0\right) + \frac{1}{R^2} - \beta_0 \left(\begin{smallmatrix}\leq\\=\\>\end{smallmatrix}\right) 0 \quad \text{if} \quad R \left(\begin{smallmatrix}\geq\\=\\<\end{smallmatrix}\right) \phi(Z) \end{aligned}$$

where $\phi(Z) = \sqrt{\frac{Z}{1 - Z(1-\beta_0 Z)}}$ is a continuous function that crosses the frontier with region 3 in $R = R_{SN}$ and the frontier with region 1 in $R = 1/\sqrt{\beta_0}$.

Since $\partial\Pi_2/\partial Z < 0$, the local maximum lies in the western frontier of region 2 ($Z = 1/\bar{\beta}$) [18], whereas the sign of $\partial\Pi_2/\partial R$ implies that the optimal interest rate is $R_{SN} = \phi(1/\bar{\beta})$ (see figure 16).

Now we examine region 3. The partial derivatives are :

$$\begin{aligned} (1-\beta_0)\,\frac{\partial \Pi_3(R,Z)}{\partial Z} &= \frac{R-1}{Z^2} > 0 \\ (1-\beta_0)\,\frac{\partial \Pi_3(R,Z)}{\partial R} &= \frac{1}{R^2} - \frac{1}{Z} < 0 \quad \text{since} \quad R > \sqrt{Z} \end{aligned}$$

[18] The local maximum can't lie in the line $R = Z$ since $\partial\Pi_2/\partial R < 0$ when $R = Z$.

R and Z are therefore pushed toward the frontiers with region 4 and 2 (see figure 16). Note that the profit function jumps upward when the frontier between region 3 and region 2 is crossed, since :

$$\Pi_2(R,1/\bar{\beta}) - \Pi_3(R,1/\bar{\beta}) = \frac{\bar{\beta}-\beta_0}{1-\beta_0}\left(\frac{R}{\bar{\beta}}-1\right) > 0$$

This implies that the optimum can't lie in region 3.

Now let's examine region 4 and 5. The partial derivatives are :

$$\begin{aligned}
(1-\beta_0)\frac{\partial \Pi_4(R,Z)}{\partial Z} &= \frac{R-1}{Z^2} > 0 \\
(1-\beta_0)\frac{\partial \Pi_4(R,Z)}{\partial R} &= 2\left(\frac{1}{R^3}-\beta_0\right)+\frac{1}{R^2}-\frac{1}{Z}\,(\lesseqgtr)\,0 \quad \text{if} \quad Z\,(\lesseqgtr)\,\psi(R) \\
(1-\beta_0)\frac{\partial \Pi_5(R,Z)}{\partial Z} &= -\frac{R(R-1)}{Z^2} < 0 \\
(1-\beta_0)\frac{\partial \Pi_5(R,Z)}{\partial R} &= (2R-1)\left(\frac{1}{Z}-\beta_0\right)+\frac{1}{R^2}-\frac{1}{Z} > 0 \quad \text{since} \quad R \leq \sqrt{Z}
\end{aligned}$$

where $\psi(R) = \left(2\left(\frac{1}{R^3}-\beta_0\right)+\frac{1}{R^2}\right)^{-1}$ is an increasing function of R which cross the $R = Z$ line in $Z = Z_1 \in]1,(1/\beta_0)^{2/3}[$ and the frontier between region 4 and 5 in $R = (1/\beta_0)^{1/3}$ and $Z = (1/\beta_0)^{2/3}$.

The sign of the derivatives imply that the local maximum lies in the frontier between region 4 and 5. It is easy to check that the local optimum is such that : $R = (1/\beta_0)^{1/3} = R_S$ and $Z = (1/\beta_0)^{2/3} = Z_S$ (see figure 16) [19].

Now we show that strategy (S) is chosen for low $\bar{\beta}$ whereas strategy (SN) is optimal otherwise. Let's compute the optimal profit for both strategies :

$$\begin{aligned}
\Pi_S &= \Pi_4(R_S,Z_S) = \frac{1-\beta_0^{1/3}}{1-\beta_0}\left(2-\beta_0^{1/3}-\beta_0^{2/3}\right) \\
\Pi_{SN} &= \Pi_2(R_{SN},Z_{SN}) = \frac{1}{1-\beta_0}\left(1+\beta_0-\frac{2}{R_{SN}}\right)
\end{aligned}$$

Thus $\Pi_S > \Pi_{SN}$ when :

$$R_{SN} = \sqrt{\frac{\bar{\beta}}{\beta_0-\bar{\beta}(1-\bar{\beta})}} > \frac{2}{3\beta_0^{1/3}-1}$$

which is verified for all $\bar{\beta} \in [\beta_0,\bar{\beta}_1[$, where $\bar{\beta}_1 = \frac{-b-\sqrt{b^2-4ac}}{2}$ (with $a = 1$, $b = -(1+(3\beta_0^{1/3}-1)^2/4)$ and $c = \beta_0$) is the unique positive solution of the equation :

$$\bar{\beta}^2 - \left(1+\frac{1}{4}\left(3\beta_0^{1/3}-1\right)^2\right)\bar{\beta}+\beta_0 = 0$$

Now we compute the upper bound $\bar{\beta}_2$. The strategy (SN) is chosen when $\bar{\beta} \geq \bar{\beta}_1$ and :

$$R_{SN} \leq Z_{SN} = 1/\bar{\beta} \quad \Longleftrightarrow \quad \sqrt{\frac{\bar{\beta}}{\beta_0-\bar{\beta}(1-\bar{\beta})}} \leq 1/\bar{\beta}$$

$\bar{\beta}_2$ therefore solves the following cubic equation :

$$\Upsilon(\bar{\beta}) = \bar{\beta}^3 - \bar{\beta}^2 + \bar{\beta} - \beta_0 = 0$$

It is easy to check that equation $\Upsilon(\bar{\beta}) = 0$ has a unique solution in the interval $[\beta_1, 1]$. [20]

[19] The optimum strategy is $R_S = \operatorname{argmax} \Pi_4(R,R^2)$ and $Z_S = R^2$.

[20] $\Upsilon(\beta_0) = \beta_0^2(\beta_0-1) < 0$, $\Upsilon(1) = 1-\beta_0 > 0$ and $\Upsilon'(\bar{\beta}) = 3\bar{\beta}^2 - 2\bar{\beta} + 1 > 0$, $\forall \bar{\beta} \in [\beta_0, 1]$

Case 2: $\beta_0 \leq \bar{\beta} < \sqrt{\beta_0}$**,** $R_{SN} = \sqrt{\frac{\bar{\beta}}{\beta_0 - \bar{\beta}(1-\bar{\beta})}} \geq \frac{1}{\bar{\beta}}$ **and** $R_N < 1/\bar{\beta}$

The optimum profit strategy is :

$$\text{(NS)} \qquad R = 1/\bar{\beta} = R_{NS} \quad \text{and} \quad Z = 1/\bar{\beta} = Z_{NS} \quad \text{for} \quad \bar{\beta}_2 \leq \bar{\beta} < \bar{\beta}_3$$

where $\bar{\beta}_3$ is given at the end of the subsection.

Proof.

As before (see figure 17) there is a local optimum at the frontier between region 4 and 5 ($R = R_S$ and $Z = Z_S$). We shall prove that this optimum can't be the global one

The other local optimum lies either at the western frontier or at the northern frontier of region 2 (since $\partial \Pi_2 / \partial Z < 0$). The profit maximizing strategy at the western frontier (the one such that $Z = 1/\bar{\beta}$) is $R = Z = 1/\bar{\beta}$ since $\partial \Pi_2(R, 1/\bar{\beta})/\partial R > 0$. This strategy is also the profit maximizing strategy at the northern frontier since $R_N < 1/\bar{\beta} \Rightarrow d\Pi_2(R,R)/dR < 0 \forall R \geq 1/\bar{\beta}$. [21]

Therefore $\bar{\beta}_3 = 1/R_N$ (since $R = Z = 1/\bar{\beta}$ is no longer the profit maximizing strategy when $R_N \geq 1/\bar{\beta}$).

Let's show that the (NS) strategy is better than the (S) strategy when $\bar{\beta} \in [\bar{\beta}_2, \bar{\beta}_3[$. The profit of the (NS) strategy writes :

$$\Pi_{NS} = \Pi_2\left(1/\bar{\beta}, 1/\bar{\beta}\right) = \frac{\bar{\beta} - \beta_0}{1 - \beta_0}\left(\frac{1}{\bar{\beta}^2} - 1\right)$$

Therefore :

$$\frac{d\Pi_{NS}}{dR} = \frac{1}{1 - \beta_0}\left(-\frac{1}{\bar{\beta}^2} + 2\frac{\beta_0}{\bar{\beta}^3} - 1\right)$$

It is easy to show that $d\Pi_{NS}/d\bar{\beta} > 0$ if $\bar{\beta}_2 \leq \bar{\beta} < 1/R_N$ [22]. This and $\Pi_{NS} = \Pi_{SN} > \Pi_S$ for $\bar{\beta} = \bar{\beta}_2$ imply that $\Pi_{NS} > \Pi_S \; \forall \bar{\beta} \in [\bar{\beta}_2, \bar{\beta}_3[$.

Case 3: $R_N \geq 1/\bar{\beta}$ [23]

The optimum profit strategy is :

$$\text{(N)} \qquad R = R_N \quad \text{and} \quad Z = Z_N \quad \text{for} \quad \bar{\beta}_3 \leq \bar{\beta} \leq 1$$

Proof.

As seen in figure 18, there is a local optimum at the frontier between region 4 and 5 and another at the northern frontier of region 2 (since $d\Pi_2/dR > 0$). The profit function in the latter case writes:

$$\Pi_N = \Pi_2(R, R) = \left(R^2 - 1\right) G\left(1/R^2\right)$$

which is also the profit when the agents are fully naïve. The unique maximum of Π_N is attained when $d\Pi_N/dR = 0 \Leftrightarrow R = R_N$ (see section B for the full proof).

[21] Recall that R_N is the profit maximizing strategy for fully naïve agents.

[22] Let $d\Pi_{NS}/d\bar{\beta} = \Omega(\bar{\beta})$. Simple algebra yields $\Omega(\bar{\beta}_2) = -\frac{\bar{\beta}_2^2 - \beta_0}{1 - \beta_0} > 0$ (since $\bar{\beta} < \sqrt{\beta_0}$) and $\Omega(\bar{\beta}_3) = 0$ (since $\bar{\beta}_3 = 1/R_N$ and $-2\beta_0 R_N^3 + R_N^2 + 1 = 0$). This and the fact that $\bar{\beta}_3$ is the only solution of the the equation $\Omega(\bar{\beta}) = 0$ in the interval $[\beta_0, 1]$ implies that $\Omega(\bar{\beta}) > 0$, $\forall \bar{\beta} \in [\bar{\beta}_2, \bar{\beta}_3[$

[23] There a two sub-cases : case 3.a (such that $\bar{\beta} < \sqrt{\beta_0}$ and $R_N > 1/\bar{\beta}$) and case 3.b ($\bar{\beta} \geq \sqrt{\beta_0} \Rightarrow R_N > 1/\bar{\beta}$). Figure 18 represents case 3.b

The (N) strategy is always better than the (S) strategy since $\Pi_N \geq \Pi_{NS} > \Pi_{SN} > \Pi_S$.

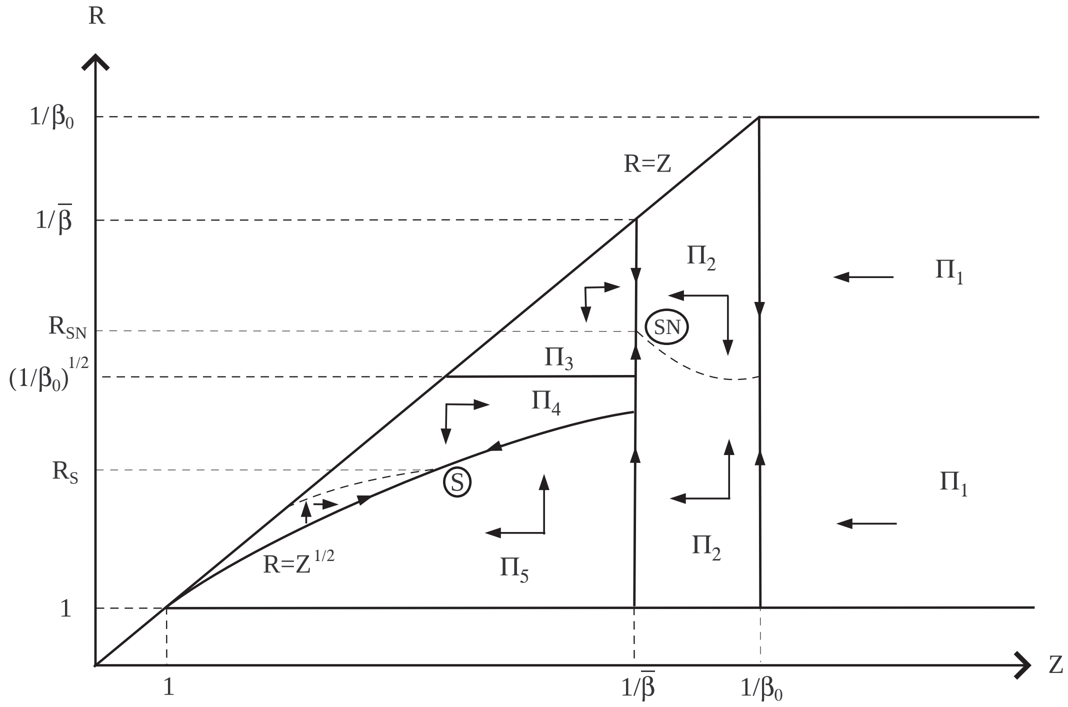

Figure 16: A mapping of the profit function when $\beta_0 < \bar{\beta} < \sqrt{\beta_0}$ and $R_{SN} < 1/\bar{\beta}$

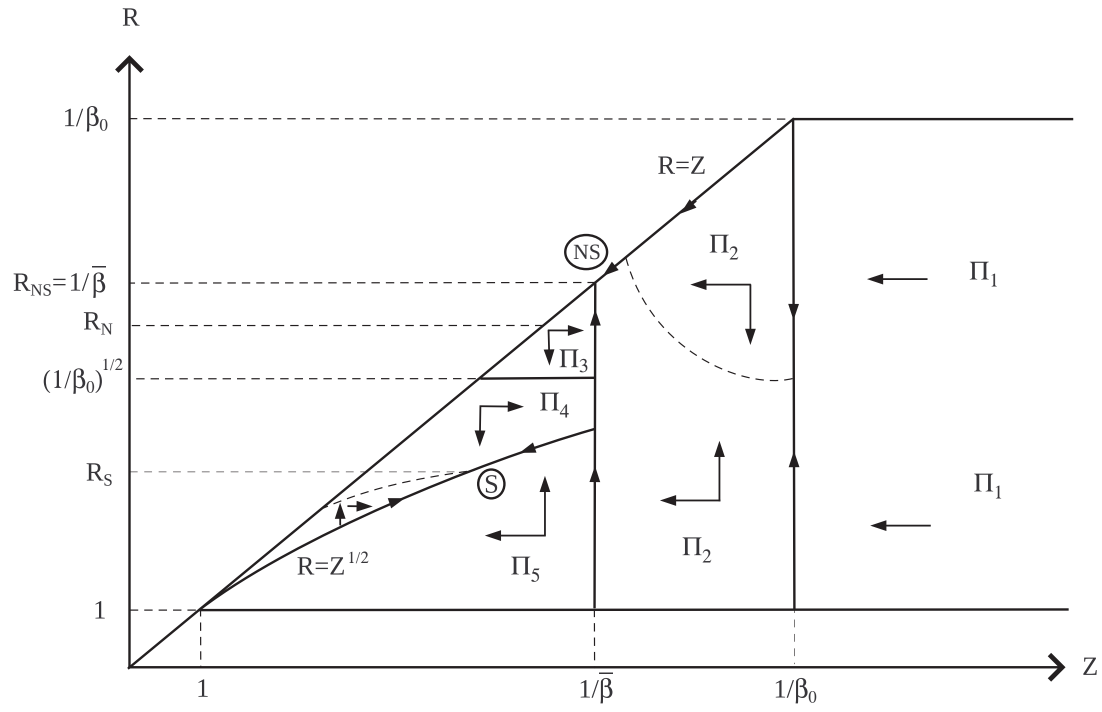

Figure 17: A mapping of the profit function when $\beta_0 < \bar{\beta} < \sqrt{\beta_0}$, $R_{SN} \geq 1/\bar{\beta}$ and $R_N < 1/\bar{\beta}$

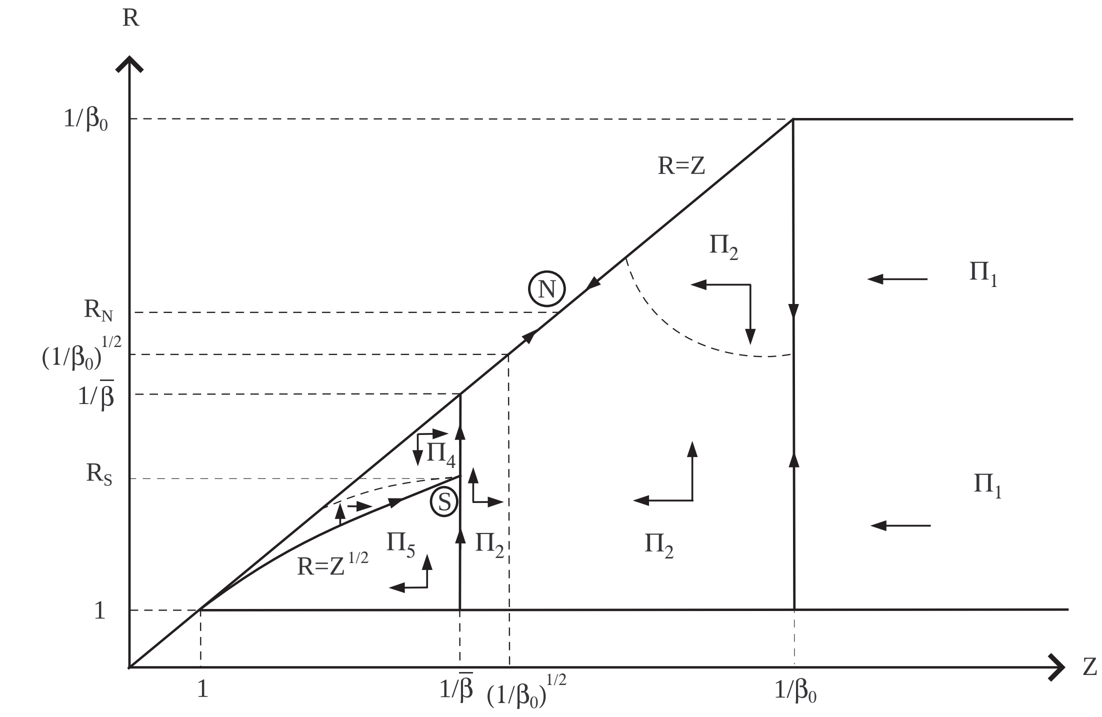

Figure 18: A mapping of the profit function when $\beta_0 < \sqrt{\beta_0} \leq \bar{\beta}$ and $R_N > 1/\bar{\beta}$

D The impact of competition

D.1 Fully naïve borrowers

The equilibrium is such that : (i) all firms choose the same R ; (ii) each firm chooses F so as to maximize her expected profit given R ; (iii) expected profits are null. Assuming that there is a continuum of firms of size m (the variable m is unimportant), condition (ii) implies :

$$F^*(R) = \arg\max_F \Pi_i(R,F) = \frac{1}{m}\left[(R(R+F-1)G\left(\frac{1}{R+F}\right) + (R-1)\right]$$

where $\Pi_i(R,F)$, the profit of firm i, is a concave function of F. Thus :

$$\frac{\partial \Pi_i(R,F)}{\partial F} = \frac{1}{m}\frac{R}{1-\beta_0}\left(\frac{1}{(R+F)^2} - \beta_0\right) = 0 \quad \Longrightarrow \quad F^*(R) = \frac{1}{\sqrt{\beta_0}} - R$$

The zero profit condition (iii) then writes :

$$\Pi_i(R, F^*(R)) = \frac{1}{m}\left[\frac{2R}{1+\sqrt{\beta_0}} - 1\right] = 0$$

which yield $R_N^c = (1+\sqrt{\beta_0})/2$ and $F_N^c = F^*(R_N^c)$.

We use these calculations to show that the expected profit of a firm that chooses $R < 1$ and $F > 0$ when all other firms choose $R = 1$ is positive for some F. Let $R = 1-\epsilon$ and $F^*(1-\epsilon)$ be the interest rate and the fee chosen by the firm. Her profit writes :

$$\Pi_i(1-\epsilon, F^*(1-\epsilon)) = \left[\frac{2(1-\epsilon)}{1+\sqrt{\beta_0}} - 1\right]$$

which is positive $\forall\ \epsilon < (1-\sqrt{\beta_0})/2$.

D.2 Partially naïve borrowers

The fee is chosen so as to maximize the expected profit per consumer subject to condition (18) :

$$F^{**}(R) = \arg\max_F \Pi_i(R,F) = \frac{1}{m}\left[(R(R+F-1)G\left(\frac{1}{R+F}\right) + (R-1)\right] \quad \text{s.t.} \quad R+F \geq 1/\bar{\beta}$$

This program has two solutions : constrained and unconstrained. The unconstrained solution is the one given in appendix D.1. It arises when $R + F^* = 1/\sqrt{\beta_0} \geq 1/\bar{\beta}$.

The constrained solution is such that : $R + F^{**} = 1/\bar{\beta}$ and $\frac{\partial \Pi}{\partial F} < 0$ (which is a consequence of $R + F^{**} = 1/\sqrt{\beta_0} < 1/\bar{\beta}$). Then the profit writes :

$$\Pi_i(R, F^{**}(R)) = R\left(\frac{1}{\bar{\beta}} - 1\right)\frac{\bar{\beta}-\beta_0}{1-\beta_0} + R - 1$$

The zero profit condition $\Pi_i(R, F^{**}(R)) = 0$ yield the equilibrium value R_{PN}^c and $F_{PN}^c = 1/\bar{\beta} - R_{PN}^c$.

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